Abstract

The electoral success of an extremist party usually attracts considerable attention. Yet the rarely have an opportunity to directly influence policy as they are, more often than not, shut out of the policy making process by mainstream parties. Extremist parties may, however, influence policy indirectly by inducing mainstream parties to adjust their electoral strategies. I consider a model of electoral competition between an expressive extremist party and two mainstream parties in first-past-the-post and majority runoff elections. The presence of an extremist party results in an equilibrium policy outcome that is further away from the extremist’s preferred policy. The magnitude of the effect is also shown to depend on the type of majoritarian electoral system.
1 Introduction

Much of the literature on extremist parties has been concerned with the rise of extremist parties and the conditions under which extremist parties emerge and become successful (see, e.g., Carter, 2005; Norris, 2005; Arzheimer & Carter, 2006; Mudde, 2007).1 Less effort has been devoted to examining the political consequences of extremist parties. Extremist parties, where they have been successful, have by-and-large been excluded from government participation. When they are allowed to take part, havoc normally ensues. The EU countries reacted harshly to the formation of a coalition between the Austrian Freedom Party and the People’s Party in 2000, and threatened to boycott the government politically. In France, the cooperation between the centre-right and the National Front at the regional level was frowned upon by national party organization and eventually led to an electoral reform at the regional level designed to create large majorities in order to remove the temptation of the regional parties to look to the National Front for support.

If extremist parties are marginalized when it comes to forming government coalitions as well as in the policy making process one might assume that extremist parties simply have no influence on policy. If that is the case the attention devoted to extremist in both journalistic and scholarly accounts would appear to be much ado about nothing. While the discussion above suggests that extremist parties have no direct means of influence policy outcome, their presence may nevertheless influence policy as other political actors are forced to react to the presence of the extremist parties. The presence of extremist parties invariably affects political discourse and, as a result, mainstream political parties have been shown to react to the emergence of extremist parties (Meguid, 2005, 2010). In other words, extremist parties influence policy outcomes indirectly through the effects they have on electoral competition. Their presence forces mainstream parties to alter their campaign strategies and, in particular, to adopt different electoral platform.

There appears to be a general sense that the emergence of an extremist party has negative consequences, e.g., that greater prevalence of extremists views will shift policy away from the center and towards the extremist party’s preferred policies. However, as Meguid (2010) convincingly argues, mainstream political parties have options — they can always choose to ignore the extremist party.

1See, e.g., Kitschelt (2007) for a useful overview of the recent literature.
While ignoring the extremist party might be socially optimal, the mainstream parties are in competition with one another and the presence of an extremist party affects their competition. The emergence of the Tea Party in the U.S. is a recent example that nicely highlights how extremist parties affect the competition between the mainstream parties. The presence of Tea Party candidates forced mainstream Republicans to reconsider their strategy. Ignoring the presence of the Tea Party candidates would leave open the possibility of losing the primary while moving to the right in order to neutralize Tea Party candidates could potentially disadvantage the mainstream Republicans in the general election. The mainstream Republicans’ decisions obviously affect the Democrats as well. If a mainstream Republican adopts a platform further to the right, the Democratic candidate will become more attractive to centrist (and even center right) voters and her chances of winning are improved. This, in turn, may affect the Democratic candidate’s strategy — especially if the mainstream Republican moves far to the right. Because the mainstream Republican candidate is moving to the right, the Democratic candidate may be able to adopt a more leftist position and still win the election. Thus, mainstream Republicans must also take into account how their strategic choices influence the behavior of their opponents.

In this paper I examine how these strategic considerations shape the choice of electoral platforms in plurality and majority runoff elections and, subsequently, how policy outcomes are affected by the presence of an extremist party. To foreshadow my conclusions, the results are somewhat counterintuitive: The presence of an extremist party does not move policy outcomes closer to the extremist party but, rather, moves policy further away from its preferred policy.

2 Extremist Parties & Policy

The electoral success of extremist parties invariably attracts considerable attention. As extremist parties rarely have the opportunity to make policy or even to influence it much, all the attention heaped upon extremist parties appears to be unwarranted or have to do with factors besides political outcomes, e.g., concern about the prevalence of extremist views in society. But it is also possible that extremist parties affect policy outcomes despite the fact that they tend to be

\[^{2}\text{Of course, the Tea Party is not a proper party and, in addition, the presence of party primaries changes the game slightly but the case nevertheless nicely illustrates the mainstream parties’ strategic considerations.}\]
marginalized in the policy making process.

Some models of electoral competition in parliamentary systems implicitly assume extremist parties influence policy outcomes. These models assume that the implemented policy outcome equals the vote-weighted average of the parties’ policy platforms. Thus, the support of extremist parties has a direct impact on policy outcomes, providing both extremist candidates with an incentive to run and voters to vote for them. Unfortunately, as De Sinopoli & Iannantuoni (2007) show, it is not only rational for extremist voters to vote for extremist candidates, it is also rational for centrist voters to vote for extremist parties. By doing so, the centrist voters can edge the policy outcome slightly to the left or the right — the only centrist voters that don’t have an incentive to vote for extremist parties are those who get their most preferred policy implemented. In equilibrium, then, only the two most extreme parties receive votes although the policy outcome will be centrist. Kedar (2005) argues that this logic accounts for the tendency of voters to vote for parties that have more extremist preferences than they do themselves. In her view, these incentives are, however, tempered, by expressive preferences, i.e., that voters don’t only care about policy outcome but also about voting for parties that they find agreeable.

The problem with the above models is that they abstract away one of the most crucial feature of parliamentary systems. Legislatures are by-and-large majoritarian creatures and legislative powers are largely invested in the hands of a government that, explicitly or implicitly, has the support of a majority of the legislature (Indridason, forthcoming). In other words, the legislative power of opposition parties in parliamentary systems is generally negligible. While there are instances in which extremist parties have been brought into government, thus allowing them direct access to legislative influence, those instances can be considered rare. The focus here will, therefore, be on analyzing how extremist parties influence policy outcomes indirectly through the strategic incentives the create for mainstream political parties.

2.1 Competing with Extremist under FPTP

I start by considering a simple model of electoral competition between a party driven by expressive motives and two instrumental parties who have preferences

\[3\text{See, e.g., Kedar (2005) and Ortuño-Ortín (1997).}\]
\[4\text{Mudde (2007) points out that mainstream parties haves sometimes required far right parties to sign agreements not to implement parts of their electoral platforms.}\]}
over government policy. The expressive party derives its utility from simply taking a stand, i.e., running for office on its most preferred platform. If the expressive party’s most preferred platform is extreme we can think of the party as an extremist party that contests the election without regard to its potential of winning office.\(^5\) There is wealth of anecdotal evidence that suggest that certain parties and candidates are driven by expressive motives rather than instrumental ones. There is also some systematic evidence suggesting that extremist and niche parties behave as expressive parties. Adams et al. (2006), e.g., find that niche parties do not adjust their policy platforms in response to changes in public opinion.\(^6\)

The two policy-seeking parties may have extreme policy preferences but what distinguishes them from the expressive party is that they care about winning elections in order to implement their preferred policy outcome rather than simply taking a stand. While the model can accommodate any policy preferences, the most empirically relevant scenarios are the ones in which two relatively centrist parties compete against an extremist party.

Formally, let the set of parties be \(\{0, 1, 2\}\). The set of outcomes of the game is a policy in the unidimensional policy space \([0, 1]\) and each parties’ most preferred policy outcome is denoted \(x^k\). Without loss of generality it is assumed party 0 has expressive preferences and that \(0 = x^0 \leq x^1 \leq x^2\).\(^7\) The set of preference profiles is assumed to be single-peaked on \([0, 1]\). That is, for expressive candidates, the payoff is decreasing in the distance between their ideal policy and their announced platform and, for non-expressive candidates, each candidate’s payoff is decreasing in the distance between her ideal policy and the implemented policy.

Voters’ preferences over policy are defined in an analogous manner to the candidate preferences and the distribution of the voters’ of ideal policies is described by the cumulative distribution function \(F(x)\). For simplicity voters’

\(^5\)It is also possible to think of expressive parties as parties that primarily make appeals to voters on expressive grounds, i.e., that the party is a representative of a particular groups of voters. Schuessler (2000), among others, argues that voters are driven by expressive motives and, as Mair (2009) points out, expressive voters require expressive parties. Golder (2003) finds that the support of neofascist parties doesn’t vary with socioeconomic factors and electoral system constraints as would be expected if their voters were instrumentally motivated.

\(^6\)As the most interesting case for study is that of expressive extremist parties, I use the two labels interchangeable.

\(^7\)While much of the interest in extremist parties may be focussed on far right parties there is a slight notational advantage in making the extremist party a left party. Analogous results hold if \(x^0 = 1\). In appendix B, I also show that the main substantive conclusions remain if the extreme is assumed to be less extreme (i.e., \(x^0 \neq 0 \neq 1\)).
ideal policies are assumed to be distributed according to the uniform distribution.

The game consists of two stages. At the first stage, each candidate chooses a policy platform, \( p_k \in [0, 1] \). At the second stage, each voter \( i \) casts a vote for one of the candidates, \( a_i \in \{0, 1, 2\} \). Under first-past-the-post electoral system, the candidate that receives the most votes wins the election and implements her policy platform. That is, it is assumed that candidates can credibly commit to their policy platform. Voters are assumed to cast their votes sincerely, i.e., \( a_i = k \) only if \( u_i(p_k) \geq u_i(p_j), \forall j \neq k \). The consequences of relaxing this assumption are considered in the appendix.

As it turns out, a pure strategy Nash equilibrium doesn’t always exist in the game. I solve for Nash equilibria when they exist but when they don’t I adopt a slightly modified version of a Nash equilibrium in which the candidates don’t consider very small deviations from their strategies. That is, given a strategy profile in which candidate \( k \) chooses platform \( p^k \) then the candidate will not consider deviations that involve the strategies in \( [p^k - \varepsilon, p^k + \varepsilon] \). Intuitively one can think of this meaning that voters don’t pay attention to very small changes the party’s policy platform or simply that there are limits to the parties abilities to construct arbitrarily small changes in their platforms. I term this equilibrium concept \( \varepsilon \)-Nash equilibrium. The requirement is weak because \( \varepsilon \) can be arbitrarily small.\(^8\)

For sake of comparison it is instructive to consider the outcome of the game in the absence of an expressive candidate. This result is familiar from the literature (see, e.g., Osborne, 1995). The median voter plays an important role in the competition between policy motivated candidates much as is the case with office-seeking candidates. In short, the median voter determines the policy outcome in a simple manner. If the two candidates’ ideal policies are located at opposite sides of the median voter the equilibrium policy in the game corresponds to the median voters preferred policy. If the two candidates’ ideal policies are located at the same side of the median voter the equilibrium policy corresponds to the ideal policy of the candidate that is closer to the ideal policy of the median voter. It is a simple matter to verify that the \( \varepsilon \)-Nash equilibrium policy outcomes, in the limit as \( \varepsilon \) tends to zero, are identical.

The presence of an extremist candidate changes the dynamics of the competi-

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\(^8\)Another way to solve the equilibrium non-existence problem would be to simply assume that the parties must differentiate themselves by not adopting platforms that are too similar. The approach used here is similar to an \( \varepsilon \)-equilibrium in which players ignore deviations that increase their payoff by less than \( \varepsilon \) (Radner, 1980).
tion. In particular, it is no longer true that the median voter holds the privileged position described above. Instead, if the expressive candidate’s most preferred policy is \( x^0 = 0 \), the voters located at \( \frac{2}{3} \) will often be in a pivotal position.

Before we derive the equilibria of the game it is useful to provide some additional notation. I use \( p^*_k \) to denote candidate \( k \)’s equilibrium strategy. For any policy \( p_k \), define \( \hat{p}_k \) and \( \bar{p}_k \) such that \( |\frac{2}{3} - p_k| = |\frac{2}{3} - \hat{p}_k| = |\frac{2}{3} - \bar{p}_k| \) and \( \bar{p}_k < \hat{p}_k \). It will be shown that in the cases where the voters with ideal policies located at \( \frac{2}{3} \) are pivotal, candidate \( j \) can beat candidate \( k \) by choosing a platform in \( W_j(p_k) = (\hat{p}_k, \bar{p}_k) \).

It is convenient to additionally define the set of positions that can be reached by the smallest possible change (\( \varepsilon \)) in a party’s policy platform from inside the winset of \( p_k \), \( W_j^{\varepsilon}(p_k) \). Let \( W_j^{-\varepsilon}(p_k) \) denote the set of positions that be can reached by moving to the left. Thus, \( W_j^{\varepsilon}(p_k) = (p_k + \varepsilon, \bar{p}_k + \varepsilon) \) and \( W_j^{-\varepsilon}(p_k) = (p_k - \varepsilon, \hat{p}_k - \varepsilon) \).

The intersection \( W_2(p_1) \cap W_2^{\varepsilon}(p_1) \) then represents the platforms \( p_2 \in W_2(p_1) \) such that there exist a deviation \( p'_2 > p_2 + \varepsilon \) which is still winning. Consequently any platform \( p_2 \in W_2(p_1) \cap W_2^{\varepsilon}(p_1) \) cannot be an equilibrium strategy if candidate 2 prefers a policy further to the right. That is, if \( p_2 \in W_2(p_1) \cap W_2^{\varepsilon}(p_1) \) there exists another policy in the winset that candidate 2 prefers more. On the other hand, any \( p_2 \in W_2(p_1) \setminus (W_2(p_1) \cap W_2^{-\varepsilon}(p_1)) \) offers no opportunities for profitable deviations. Similarly, \( p_1 \in W_1(p_2) \setminus (W_1(p_2) \cap W_1^{\varepsilon}(p_2)) \) represent the platforms that candidate 1 cannot deviate from to a platform further to the left without losing the election.

I start by considering what the expressive candidate’s prospects of winning.

**Lemma 1** Candidate 0 loses the election if \( p_1 \neq p_2 \).

**Proof:** Suppose \( p_1 < p_2 \). Then candidate 0’s vote share, \( v_0 \), equals \( \frac{p_1}{2} \). The vote shares of the other candidates are \( v_1 = \frac{p_1 + p_2}{2} - \frac{p_1}{2} = \frac{p_2}{2} \) and \( v_2 = 1 - \frac{p_1 + p_2}{2} \). Thus, \( v_1 > v_0 \) by \( p_1 < p_2 \). \( \square \)

Lemma 1 shows that the expressive candidate never wins as long as the other two candidates don’t adopt the same platform. It is a simple matter to verify that the expressive candidate can win if the other candidates do adopt the same platform. Suppose for example that candidates 1 and 2 locate at 1. In this case candidate 0 receives half the vote, while candidates 1 and 2 split the other
half between them. Thus, unless the voters are able to coordinate their actions, candidate 0 wins.

Lemma 1 also shows that the extremist party in this model is a marginal party in the sense that its hopes of winning the elections are, a priori, almost non-existent. The assumptions of the model can be considered strict in this sense but as we shall see, this does not imply that such parties are irrelevant for the outcome of the elections.

It is worthwhile briefly considering how relaxing the assumptions might alter Lemma 1. Assuming a uniform distribution of voters is arguably a strict assumption. However, Lemma 1 holds for any distribution that is symmetric about .5 with \( f'(x) > 0, \forall x < .5 \). The assumption that \( x^0 = 0 \) is more crucial here. More action profiles result in candidate 0 winning if \( x^0 \) increases. Thus, our results here apply perhaps most clearly to expressive extremist parties. The effects of less extremism are, however, a matter of degree and don’t result in qualitatively different results. Somewhat counterintuitively, less extremism leads to more moderate positions by the other parties.\(^9\)

The equilibria of the game are, not surprisingly, contingent on the policy preferences of the two parties that are motivated by policy. The equilibria are most easily characterized by focusing on distinct scenarios that are defined by three intervals in which the parties’ preferred policies may be located. I begin by considering the scenario in which neither party is a right extremist, i.e., their preferred policies lie in the interval \([0, \frac{2}{3}]\). Subsequently I turn to the scenarios where one or both the parties’ preferred policies can be characterized as extreme right policies, i.e., greater than \( \frac{2}{3} \). Multiple equilibria exist in most instances because of the assumption that a deviation must change the policy platform by at least \( \varepsilon \). As this is primarily a technical assumption, I will generally focus on the limit of the equilibrium strategies and outcomes as \( \varepsilon \) tends to zero.\(^10\)

Proposition 1 characterizes the equilibria of the game when both of the policy seeking parties’ preferred policies are to the left of \( \frac{2}{3} \). In this scenario candidate 1’s position is in a certain sense weak because candidate 1 will lose if he adopts a platform anywhere between 0 and \( x^2 \) — provided that candidate 2’s platform

\(^9\)See appendix B for more details.
\(^10\)For the same reason I also ignore the ‘effect’ of \( \varepsilon \) on the suppositions of each proposition. That is, the suppositions should also be thought of as the inequality statements as \( \varepsilon \) tends to zero although for non-zero values of \( \varepsilon \) the proposition may also hold, or fail to, in a small neighborhood of the threshold. While stating the suppositions is straightforward it clutters the statement of the results without adding anything of substantive interest. The more detailed statement can be requested from the author.
doesn’t lie to the right of $\frac{2}{3}$. Moreover, candidate 1 will rarely gain from choosing a platform to the right of $x^2$. The reason the candidate might want to do so is if he preferred the platform 0 to $x^2$. However, because $x^2 < \frac{2}{3}$ choosing such a platform is no longer sufficient to make candidate 0 the winner (again, if $p_2 < \frac{2}{3}$). Because the ideal policy of the pivotal voter is located in a subset of the policy space $(x^2, 1)$ that contains neither of the candidates’ ideal platforms a Nash equilibrium exists.

**Proposition 1** Suppose $x^1 < x^2 \leq \frac{2}{3}$. Any strategy profile $p^* = (0, p^*_1, x_2)$ where $p^*_1 \in [0, x^2) \cup (1 - \frac{x^2}{2}, 1]$ and $p^*_2 \in (x^2, x^2)$ is a Nash equilibrium of the game.

**Proof:** Candidate 0’s strategy is optimal as $x^0$ is the platform that maximizes her utility. Candidate 2’s strategy is optimal as the outcome of the game when the players take the actions described in the proposition is $x^2$, candidate 2’s most preferred policy. Finally, $p^*_1$ is optimal because only if candidate 1 deviates to $p'_1 \in (x^2, 1 - x^2)$ does candidate 1 win. By $x^1 < x^2$, candidate 1 will be made worse of by any such deviation. □

In this first scenario where neither party holds extreme right policy preferences, candidate 2 always locates within $\varepsilon$ of her ideal policy while candidate 1 locates either to the left of candidate 2 or sufficiently far to the right to ensure that candidate 2 wins the election. Figure 1 graphs the possible equilibria of the game. Candidate 2 adopts its preferred platform $x^2$ while any platform on the shaded section of the policy space is an equilibrium platform for candidate 1. If it were assumed that candidate 1 receives a small expressive payoff from adopting its most preferred platform then there would exist a unique equilibrium (under the conditions of the proposition) with $p^*_1 = x^1$.

![Figure 1: Example of an Equilibrium (Proposition 1)](image-url)
Thus, if the party furthest away from the expressive candidate is relatively moderate, that party always wins the election in equilibrium. Moreover, the winning party is able to implement its preferred policy outcome. Importantly, that policy outcome need not correspond with the median voters’ preferred policy. The policy outcome can move towards the expressive party but only if both the other parties happen to prefer policies left of the median voter — much like in the standard spatial model with two policy-seeking candidates. However, if the more right leaning candidate prefers a policy to the right of the median voter, the policy outcome moves to the right compared with what the standard spatial model predicts. In other words, Proposition 1, shows that the presence of an expressive candidate can move the policy outcome away from the expressive candidate’s preferred policy. The following propositions show that the potential for the policy outcome to move away from the expressive party is not restricted to situations where the policy-seeking parties are relatively moderate.

The next two propositions (2 and 3) characterize the equilibrium when one of the policy-seeking party prefers a moderate policy \((x^1 \leq \frac{1}{3} < \frac{2}{3})\) and the other prefers a relatively extreme right policy \((x^2 > \frac{2}{3})\). In these circumstance there exist equilibria in which each of the policy-seeking party emerges as a winner.

**Proposition 2 (Candidate 2 wins)** Suppose \(\frac{2}{3} > x^1 \geq \frac{1}{3} \) and \(x^2 \geq \frac{2}{3}\). Any strategy profile \(p^* = (0, p^*_1, p^*_2)\) where a) \(p^*_1 \in (\frac{2}{3} - \varepsilon, \frac{2}{3})\) and b) \(p^*_2 \in W_2(p^*_1) \setminus (W_2(p^*_1) \cap W_2^{-\varepsilon}(p^*_1)) \cap (p^*_1, p^*_1 + \varepsilon)\) is a \(\varepsilon\)-Nash equilibrium of the game.

**Proof:** Candidate 0’s preferences only depend on his announced platform. Choosing \(x^0\) is therefore optimal. Now consider candidate 1’s strategy. Deviation to any \(p'_1 < p^*_1 - \varepsilon\) has no effect on the outcome. Candidate 1’s vote share remains unaltered and candidate 2 wins and implements \(p^*_2\). By b) and Lemma 1 a deviation to \(p'_1 > p^*_1 + \varepsilon\) has no effect on the outcome, i.e., candidate 2 still wins. Candidate 1’s strategy is thus optimal. As candidate 2 wins under the strategy profile \(p^*\), the only question is whether candidate 2 can obtain a more favorable outcome. Since \(p^*_1 < p^*_2\) we only need to consider deviations such that \(p'_2 > p^*_2 + \varepsilon\). By b), \(p'_2 \notin W_2(p^*_1)\) and any deviation \(p'_2 > p^*_2 + \varepsilon\) would result in candidate 1 winning the election and implementing \(p^*_1\). Candidate 2’s strategy, \(p^*_2\) is optimal as \(u_2(p^*_2) > u_2(p^*_1)\). □

Figure 2 depicts the logic of proposition 2. In the figure candidate 2 wins on platform \(p^*_2\). Candidate 1’s action is optimal as deviating to any \(p'_1 < p^*_1\) doesn’t change the outcome of the election. While there exist platforms that
lead to candidate 1’s victory, e.g., $p'_1 = \frac{2}{3}$, these platforms are not obtainable as candidates only consider deviations that are larger than $\varepsilon$. Thus, any possible deviation $p'_1 > p^*_1$ would require adopting a platform greater than $p^*_2$ — in which case candidate 2 remains the winner. Similarly there exist winning platforms that candidate 2 would prefer to $p^*_2$ but the restriction to consider only changes greater than $\varepsilon$ renders those unattainable.

Thus, it should be clear that the equilibria described in proposition 2 are not Nash equilibria. This, of course, begs the question of how reasonable the restriction to $\varepsilon$-Nash equilibria is. This restriction is reasonable for two reasons. First, for the parameters of game in proposition 2, there exist no Nash equilibria. However, the players’ best responses tell us very clearly what the players’ incentives are. That is, each player has a strong incentive to locate closer to $\frac{2}{3}$ than their non-expressive competitor. Yet, locating at $\frac{2}{3}$ is not an equilibrium strategy because if both non-expressive candidates locate at $\frac{2}{3}$ then the three candidates tie and the expected payoff to each of the non-expressive candidates is lower than if they concede the election to the other non-expressive voter. However, by doing so, and choosing some other platform, it opens up

\textbf{Figure 2: Example of an Equilibrium (Proposition 1)}
profitable deviations for the candidate located at \( \frac{2}{3} \). In other words, the platform \( \frac{2}{3} \) acts as a strong center of gravity but the continuity of the policy space rules out the existence of a Nash equilibrium. Intuitively, however, one would expect the parties to locate close to \( \frac{2}{3} \).

Second, as mentioned above, the minimum deviation \( \varepsilon \) can be made arbitrarily small, which suggests that in substantive terms the assumption is not very strong. Thus, instead of \( \varepsilon \) corresponding to changing a section in a party’s manifesto it can correspond to a choice of a single adjective. In other words, it is not implausible to assume that candidates ignore such small changes to their platforms (or alternatively, that they will be lost on the voters).

Proposition 2 demonstrated the gravitational force of the platform \( \frac{2}{3} \). The equilibrium policy outcome equals \( \frac{2}{3} \) in the limit as \( \varepsilon \) tends to zero. Under proposition 2 candidate 2 wins the election with certainty. However, there also exist equilibria in which candidate 1 wins the election with certainty on a platform in the neighborhood of \( \frac{2}{3} \). Proposition 3 characterizes these equilibria.

**Proposition 3 (Candidate 1 wins)** Suppose \( \frac{2}{3} - \varepsilon > x^1 \geq \frac{1}{3} + \frac{\varepsilon}{2} \) and \( x^2 \geq \frac{2}{3} \). Any strategy profile \( p^* = (0, p^*_1, p^*_2) \) where a) \( p^*_2 \in [\frac{2}{3}, \frac{2}{3} + \varepsilon) \) and b) \( p^*_1 \in W_1(p^*_2) \cap (p^*_2 - \varepsilon, p^*_2 + \varepsilon) \) is a \( \varepsilon \)-Nash equilibrium of the game.

**Proof:** In the strategy profile \( p^* \) candidate 1 wins the election on the platform \( p^*_1 \). Candidate 0’s preferences only depend on his announced platform. Choosing \( x^0 \) is therefore optimal. By condition b), there exists no winning platform \( p^*_2 \) for candidate 1 such that \( p^*_1 < p^*_1 - \varepsilon \). Candidate 1 doesn’t prefer any deviation that results in candidate 2 winning by \( x^1 \leq p^*_1 < p^*_2 \). Thus, candidate 1’s strategy is optimal. Any deviation \( p^*_2 > p^*_2 \) only reduces candidate 2’s vote share and therefore doesn’t change the outcome of the election. A deviation \( p^*_2 < p^*_2 \) may result in candidate 2’s victory but, by \( p^*_1 \in (p^*_2 - \varepsilon, p^*_2 + \varepsilon) \), the deviation will be further away from candidate 2’s ideal policy than candidate 1’s platform, i.e., \( p^*_2 < p^*_1 \). Therefore, no profitable deviations exist for candidate 2. □

The logic behind proposition 3 is analogous to proposition 2. The only difference is that candidate 1 wins. The policy outcome in the limit as \( \varepsilon \) goes to zero remains \( \frac{2}{3} \). Next I characterize the equilibrium when candidate 1’s ideal platform is a left extremist \( (x^1 < 1) \) and candidate 2 is a right extremist \( (x^2) \).

As in propositions 2 and 3 the only possible equilibrium policy outcome is in the neighborhood of \( \frac{2}{3} \).
Proposition 4 Suppose $x^1 < \frac{1}{3}$ and $x^2 \geq \frac{2}{3}$. Any strategy profile $p^* = (0, p_1^*, p_2^*)$ such that either:

a) $p_1^* \in \left[\frac{2}{3} - \varepsilon, \frac{2}{3}\right)$ and

b) $p_2^* = (\frac{2}{3} - \varepsilon, \frac{2}{3})$

or

c) $p_1^* \in (p_2^* - \varepsilon, \frac{2}{3})$ and

d) $p_2^* = (\frac{2}{3}, \frac{2}{3} + \varepsilon)$

is a $\varepsilon$-Nash equilibrium of the game.

Proof: Candidate 0 maximizes her possible payoff by choosing the platform $x^0$. Suppose conditions a) and b) hold. A deviation to $p_1' < p_1^* - \varepsilon$ by candidate 1 doesn’t change the outcome of the election. That is, candidate 2 will still receive more than $\frac{1}{3}$ of the vote and win the election. A deviation $p_1' > p_1^* + \varepsilon$ only results in winning if $p_1' > p_2^*$ but then $u_1(p_2^*) > u_1(p_1')$. Candidate 2 wins and therefore any deviation $p_2' < p_2^*$ is not profitable. By condition a) there exist no deviations $p_2' > p_2^*$ such that candidate 2 remains the winner. Now suppose conditions c) and d) hold. By condition c), there exists no deviation for candidate 1 that results in her winning the election. Candidate 1’s strategy is therefore optimal. Candidate 2 wins and, therefore, only deviations such that $p_2' > p^*$ need to be considered. For any such deviation $\frac{2}{3} - p_1^* < p_2' < \frac{2}{3}$, by condition c) and d), in which case candidate 1 wins and implements $p_1^*$. As $u_1(p_2^*) > u_1(p_1^*)$, candidate 2’s strategy is optimal. □

It only remains to consider the situation in which candidate 1 and candidate 2 are right extremists ($\frac{2}{3} < x^1 < x^2$). As Lemma 1 shows that candidate 0 never wins unless candidates 1 and 2 adopt the same platform, predictably the policy outcome is the ideal policy of candidate 1 (the more moderate policy-seeking candidate). Thus, the policy outcome here resembles the policy outcome in the standard two candidate spatial model.11

Proposition 5 Suppose $\frac{2}{3} < x^1 < x^2$. Any strategy profile $p^* = (0, x^1, p_2^*)$ where $p_2^* \in [0, 2 - 2x^1) \cup (x^1, 1]$ is a Nash equilibrium of the game.

11The proposition focuses on the Nash equilibrium. If the $\varepsilon$-Nash equilibrium is considered then there also exists an equilibrium where candidate 2 locates close to $x^1$ and wins the election.
Proof: Candidate 0 maximizes her possible payoff by choosing the platform $x^0$. Candidate 1’s strategy is optimal as candidate 1 wins the election on her most preferred platform. Candidate 2’s strategy is optimal as the only way candidate 2 can change the outcome is to locate closer to the platform $\frac{2}{3}$ than candidate 1. Adopting a platform $p'_2 \in W_2(x^1)$ results in candidate 2’s victory but $u_2(p'_2) < u_2(x^1)$ as $p'_2 < x^1$ by $\frac{2}{3} < x^1 < x^2$. Thus, candidate 2’s strategy is optimal.

Together, propositions 1-5 characterize the equilibria of the game. To get a sense of how the presence of an expressive candidate affects policy outcomes it is instructive to compare the equilibrium outcomes obtained here with the standard model of electoral competition between policy-seeking candidates. Figure 3 graphs the policy outcome as a function of the two policy-seeking candidates’ ideal policies. Different regions of the graph are labeled with the equilibrium policy outcomes (as $\varepsilon$ tends to zero) and are shaded so that darker areas indicate policy outcomes further to the right.

Comparison of the two panels of the figure demonstrates that the observation made about Proposition 1 above more or less holds true across the possible range of ideal policies of the policy-seeking parties. The presence of a leftist expressive candidate never results in a policy outcome that is more favorable to the expressive candidate. Instead, the policy outcome is often further to the right in the presence of leftist expressive candidate.

**Figure 3: Policy Outcomes with and without an Expressive Candidate**

(as $\varepsilon \to 0$ for $\varepsilon$-Nash equilibria)

There are two circumstances in which the presence of the expressive candidate
doesn’t influence the policy outcome. If both the policy-seeking candidates prefer a policy to the left of the median voter or both prefer a policy to the right of $\frac{2}{3}$, the policy outcome that obtains is the same as when there is no expressive candidate. In all other circumstance the policy outcome is further to the right in the presence of the expressive candidate. Those circumstances include situations where the two policy-seeking parties’ ideal policies are located on the opposite sides of the median voter and where they are relatively moderate right parties. It is worthwhile noting that these are circumstances that seem more relevant empirically — i.e., it seems somewhat unlikely to observe situations where both the policy-seeking parties are relatively extreme and are located on the same end of the policy spectrum.

Thus, the consequence of an expressive (and extreme) party entering the contest has the effect of moving the policy further away from that party’s ideal policy outcome. It should be clear that if the party were assumed to care about policy and to make a strategic decision whether to enter the race then not entering would lead to a better outcome than adopting an extreme platform. The assumption of expressive position taking is arguably crucial to obtain the results above but they nevertheless highlight the policy consequences of non-competitive, or ‘non-coalitionable’, parties. In particular, it raises the question of why such parties choose to contest elections.

It also raises the question of whether these results are driven by the specific assumptions adopted here? First, the expressive candidate is assumed to be extremely extreme, i.e., her ideal policy is at the far left end of the voters’ ideal point distribution. In the appendix I show that allowing for less a extreme expressive candidate does not change the substantive finding but that the rightward policy shift is smaller. Second, the distribution of voters’ ideal policies is assumed to be uniform. Again, the results remain substantively the same for most unimodal distributions but, again, the policy effect of an expressive candidate would tend to be smaller if more voters prefer moderate or centrist policies. The results above have highlighted that the median voter is not pivotal when an expressive candidate is present. Instead, the voter located at approximately the 67th percentile is pivotal in determining the outcome of the election. Thus, the denser the distribution of voter preferences around the median voter, the smaller the rightward shift in policy will be. Third, the candidates are assumed to be purely policy-seeking. Similar results obtain when the candidates are motivated by holding office — then the equilibrium policy
outcome corresponds to the scenarios in propositions 2 and 3 remains at $\frac{2}{3}$ (assuming candidates cannot pick the same platform). It also follows that a combination of office motives and policy motives does not, in general, have much of an impact on the substantive results.

The main result, therefore, survives weakening of the assumptions about candidate location, the distribution of voter preferences, and politicians’ motives. The question why extreme candidates run, given the ‘adverse effects’ on policy, thus remains a puzzle. One factor, frequently cited in the literature as determining the success of extremist parties, is the type of electoral systems in use. In the following section we consider the effects of expressive candidates on policy outcomes in majority runoff systems.

3 Competing with Extremists: Runoff Elections

The majority runoff is a class of electoral systems although it often is identified with its most common variant in which the top two candidate in the first round of the contest advance onto the second, and final, ballot. However, majority runoff systems can vary along three dimensions: The number of ballots, the threshold for advancing onto a subsequent ballot (defined in terms of rank or vote share), and the winning requirements for each ballot (Indridason, 2008). Below we consider the two most prominent variants of the majority runoff, a top-two runoff (described above) and a two ballot runoff with a vote share threshold. Well-known examples of these electoral systems are the French presidential (top-two runoff) and legislative elections (vote share threshold).

The model in the previous section only needs slight modification to accommodate runoff elections. First, there are now two rounds of voting. We assume that the voters cast their votes sincerely on both ballots. Second, incorporating the threshold for inclusion on the second ballot is straightforward. Third, we assume that the candidates stand on the same platform in both the first and the second round of the election. The players’ action sets and payoffs are defined in the same way as before.

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12See, e.g., Harmel & Robertson (1985); Jackman & Volpert (1996); Willey (1998); Abedi (2002); Golder (2003).
3.1 The Top-two Runoff

In the top-two runoff, the two candidates receiving the most votes on the first ballot advance onto the second ballot where the candidate receiving a majority of the vote is declared the winner. The strategic situation of the policy-seeking candidates under the (top-two) runoff differs from their situation under FPTP because, first, they may care about which candidates advance onto the second ballot and, second, the absence of a third candidate on the second ballot influences the identity of the pivotal voter. In general, the effect is that candidates tend to choose more moderate positions.

The equilibrium policy outcome depends on the policy preferences of the two policy-seeking candidates in a rather predictable manner. The equilibrium of the game is characterized in the following three propositions.

**Proposition 6** Suppose $x_1 \geq \frac{1}{2}$ and $x_2 \geq \frac{1}{2}$. Any strategy profile $p^* = (0, x_1, p_2^*)$ such that $p_2^* \in (p_1^*, 1] \cup [0, 1 - p_1^*]$ is a Nash equilibrium of the game. The equilibrium policy outcome equals $x_1$.

*Proof:* Candidate 0’s strategy is optimal as she receives her maximal payoff. Candidate 1’s strategy is optimal for the same reason. For candidate 2, any deviation such that $1 - p_1^* < p_2^* < p_1^*$ will result in candidate 2 advancing onto the second ballot and subsequently winning the second ballot. Then the policy outcome is $p_2^*$ but $u_2(p_2^*) < u_2(p_1^*)$. Hence, such deviation is not profitable. $\square$

**Proposition 7** Suppose $x_1 < \frac{1}{2}$ and $x_2 > \frac{1}{2}$. The strategy profiles $p^* = (0, \frac{1}{2}, p_2^*)$ and $p^* = (0, p_1^*, \frac{1}{2})$ where $p_k^* \in [0, 1], k \in \{1, 2\}$ are Nash equilibria of the game. The equilibrium policy outcome equals $\frac{1}{2}$.

*Proof:* Candidate 0’s strategy is optimal as she receives her maximal payoff. Suppose candidate $k$ locates at $\frac{1}{2}$. Then candidate $k$ wins the election independent of the action of candidate $j \in \{1, 2\} \setminus k$. If $p_j^* < \frac{1}{2}$ candidate $k$ wins more than half the vote and advances onto the second ballot (or alternatively, is declared the winner right away) where she wins the election. If $p_j^* > \frac{1}{2}$ candidate $k$ wins more than a quarter of the vote while candidate 0 wins exactly a quarter. Candidate $k$ thus advances and wins a majority on the second ballot. If $p_j^* = \frac{1}{2}$, candidates 1 and two win $\frac{3}{8}$ of the vote each and advance onto the second ballot. In each case the policy outcome equals $\frac{1}{2}$ and thus all the candidates’ strategies are optimal. $\square$
Proposition 8 Suppose $x^1 \leq \frac{1}{2}$ and $x^2 \leq \frac{1}{2}$. Any strategy profile $p^* = (0, p^*_1, x^2)$ such that $p^*_1 \in [0, p^*_2] \cup (1 - p^*_2, 1]$ is a Nash equilibrium of the game. The equilibrium policy outcome equals $x^2$.

Proof: Candidate 0’s strategy is optimal as she receives her maximal payoff. Candidate 2’s strategy is optimal for the same reason. For candidate 1, any deviation such that $p^*_2 < p^*_1 < 1 - p^*_2$ will result in candidate 1 advancing onto the second ballot and subsequently winning the second ballot. Then the policy outcome is $p^*_1$ but $u_1(p^*_1) < u_1(p^*_2)$. Hence, such deviation is not profitable. □

The equilibria of the game should look rather familiar from our discussion of the standard model of electoral competition between two policy seeking candidates. That is, under the majority runoff the presence of an extremist candidate has no effect on the strategies pursued by the policy seeking candidates and the game is equivalent to the standard two candidate model. This is not too surprising — when the expressive party advances onto the second ballot it is always pitted against a more moderate party and is therefore sure to lose. As such the result is in some sense rather trivial.

However, this corresponds to an interesting case, i.e., the case of non-competitive or ‘non-coalitionable’ parties that normally are not expected to influence policy outcomes. Yet, as we have seen, this is not the case in first-past-the-post elections. The result here shows in a stark manner that electoral systems matter, i.e., the top-two majority runoff neutralizes non-competitive parties. It also suggests that the relationship between electoral systems and successful extremist parties is more complex than the literature suggests. That is, it may not simply be a matter of electoral systems differing in terms of their thresholds of representation, which in turn provides extremist candidates with an incentive to run and extremist voters to vote for them. The results here suggest that these decisions should also be influenced by how the decision to run influences the strategies of other candidates. Extremist candidates may be less willing to contest elections, and extremist voters may be less willing to vote for extremist candidates, if the consequence is that the policy outcome becomes less favorable to them.

3.2 Majority runoff w/vote thresholds

Some majority runoff systems do not restrict the second ballot to the top two contenders but allow any party that clears a given vote threshold to advance onto
the second ballot. Examples of such systems are the systems used for legislative elections in France and in Hungary (for the seats filled in single-member districts). In France the current threshold is 12.5% of registered voters, i.e., the actual vote share needed to advance depends on turnout in the district.\footnote{More accurately, perhaps, the threshold is defined in terms of number of votes (which doesn’t vary with turnout).} Turnout in French legislative elections is typically around 66% so in effect a party must win 18-19% of the vote to qualify for the second ballot. Hungary, on the other hand, uses a threshold rule that considers both the rank of the candidate and her vote share. The three candidates that receive the most votes advance onto the second ballot in addition to any candidate that receives more than 15% of the vote (Benoit, 1996). Here we examine majority runoff systems that only employ a vote threshold.

Denote the vote threshold by $\tau$. Majority runoff elections with vote thresholds greater than $\frac{1}{3}$ are equivalent to a top-two runoff so I restrict our attention to $\tau \leq \frac{1}{3}$. In discussing the equilibrium of in the top-two runoff I noted that the expressive party played absolutely no role and the equilibrium corresponded exactly to the equilibria of the standard two candidate model. Why might one expect the results to be any different when the threshold is defined in terms of vote shares? The reason is that the prospects of the candidate whose ideal policy lies in between the other two candidates depend on whether the expressive candidate advances onto the second ballot or not. Thus, the candidate may want to choose his platform so as to deny the expressive candidate the chance of advancing. Doing so requires the candidate to position herself closer to the expressive candidate and further away from the median position, which in turn allows the candidate at the other end of the political spectrum greater latitude in choosing a less centrist platform.

To clarify the intuition, consider a more concrete example. Suppose the vote threshold is 20% and candidates 1 and 2’s ideal policies are on the opposite sides of the median voter. Suppose candidates 1 and 2 locate at the median voter’s position. If that is the case, then candidate 0 qualifies for the second ballot with 25% of the vote. If that is the case, candidate 2’s strategy is not optimal. Since all three candidates qualify for the second ballot, candidate 2 could move to the right and win. As candidate 1’s vote share equals $v_1 = \frac{p_2}{2}$, candidate 2 could move quite far to the right as its vote share equals $v_2 = 1 - \frac{p_1 + p_2}{2}$ — thus, as it was assumed that $p_1 = .5$, candidate 2 can win by adopting any platform in
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Hence, ignoring the presence of the expressive candidate can be quite costly for candidate 1. Candidate 1 can improve its outcome by preventing the expressive candidate from qualifying for the second ballot. To do so, candidate 1 must adopt a platform slightly to the left of .4, in which case candidate 0 just fall short of the 20% threshold. This, in turn, restricts candidate 2’s ability to move to the right. Candidate 2 is still able to win the election but in order to do so she must choose a platform in the interval (.5,6). Thus, candidate 1 is made better off as long as the ideal policy of candidate 2 is located further to the right than .6.

In deriving the results for the top-two runoff it was not necessary to make explicit references to the candidates’ vote shares on the first on the second ballot but doing so is necessary to derive the following results. We use $v_{f}^{i}$ do refer to candidate $i$’s vote share on the first ballot and $v_{s}^{i}$ to refer to her vote share on the second ballot.

**Proposition 9** Suppose $x^{1} \leq \frac{1}{2}$, $x^{2} \geq \frac{1}{2}$, and $\tau \in (\frac{1}{6}, \frac{1}{3})$. There are two cases:

a) If $x^{2} \geq 1 - 2\tau$ then any strategy profile $p^{*} = (0, p_{1}^{*}, p_{2}^{*})$ such that $p_{1}^{*} \in (2\tau - \epsilon, 2\tau]$ and $p_{2}^{*} \in (1 - p_{1}^{*} - \epsilon, 1 - p_{1}^{*})$ is a $\epsilon$-Nash equilibrium of the game.

b) If $x^{2} < 1 - 2\tau$ then any strategy profile $p^{*} = (0, p_{1}^{*}, p_{2}^{*})$ such that $p_{1}^{*} \in (\frac{2}{3} - \epsilon, \frac{2}{3})$ and $p_{2}^{*} \in (p_{1}^{*}, p_{1}^{*} + \epsilon)$ is a $\epsilon$-Nash equilibrium of the game.

The equilibrium policy outcome equals $p_{2}^{*}$.

**Proof:** a) When the candidates play $p^{*}$, candidates 1 and 2 advance onto the second ballot where candidate 2 wins a majority of the vote. Candidate 0’s strategy is optimal as she receives her maximal payoff. Any deviation $p_{1}' < p_{1}^{*}$ leaves candidate 1’s vote share on the first ballot unaltered, $v_{1}' = \frac{p_{2}^{*}}{2}$, and reduces the candidate’s vote share on the second ballot to $v_{1}^{*} = \frac{p_{1}^{*} + p_{2}^{*}}{2} < \frac{p_{1}^{*} + p_{2}^{*}}{2}$. Any deviation $p_{1}' \in (p_{1}^{*}, p_{2}^{*})$ means that candidate 0 receives more than $\tau$ votes and advances onto the second ballot. Candidate 1’s vote share on the second ballot is then $v_{1}^{*} = \frac{p_{1}^{*}}{2}$. Candidate 2 receives $v_{2}^{*} = 1 - \frac{p_{1}^{*} + p_{2}^{*}}{2}$ of the vote. Since candidate 1’s vote share will always be larger than candidate 0’s vote share $\left(v_{0}^{*} = \frac{p_{2}^{*}}{2} < v_{1}^{*} = \frac{p_{1}^{*}}{2}\right)$, candidate 1’s decision hinges on whether choosing a platform close to $p_{2}^{*}$ will reduce candidate 2’s vote share sufficiently for candidate 1 to win. Such profitable deviation exists if there exist $p_{1}' \in (p_{1}^{*}, p_{2}^{*})$ such that $v_{1}^{*} = \frac{p_{1}^{*}}{2} < 1 - \frac{p_{1}^{*} + p_{2}^{*}}{2}$. The inequality reduces to $p_{1}' > 2 - 2p_{2}^{*}$ or, as $\epsilon \to 0$,
$p_1' > 2 - 2(1 - 2\tau) = 4\tau$. Since we are considering deviation such that $p_1' < p_2^*$ then it must be the case that $\tau < 1 - 2\tau$, or $\tau < \frac{1}{6}$. Hence, no such deviation exists. Finally, there are no profitable deviation such that $p_1' \geq p_2^*$. Either such deviation will not alter the outcome or candidate 1 will win but then $u_1(p_1') < u_1(p_2^*)$. Candidate 1’s strategy is thus optimal. Candidate 2 wins the contest, and is the rightmost candidate, so there are clearly no profitable deviations such that $p_2' < p_2^* - \varepsilon$. Neither are there any profitable deviations such that $p_2' > p_2^* + \varepsilon$. Given the strategies $p_1^*$ and $p_2^*$, $p_2' > p_2^* + \varepsilon$ implies that $\frac{p_1' + p_2'}{2} > .5$, i.e., candidate 1 receives the vote of the median voter on the second ballot and implements the policy $p_1^*$ but $u_2(p_1^*) < u_2(p_2^*)$. □

Proposition 9 demonstrates how, for given preference configurations, the presence of an expressive candidate prevents median convergence from taking place as occurs under the top-two runoff (given identical preferences). Figure 4 depicts an equilibrium under the conditions of proposition 9. The presence of the expressive candidate works to the advantage of the candidate at the other end of the political spectrum. However, the advantage is smaller than under FPTP because the centrist candidate (candidate 1) can choose a policy platform that ensures that she faces candidate 2 on the second ballot and, therefore, candidate 2 must adopt a platform closer to the median that ensures her a majority on the second ballot. Thus, to a certain degree, a majority runoff with vote share thresholds produces a policy outcome that lies in between the policy outcomes under FPTP and top-two majority runoff elections.

![Figure 4: Example of an Equilibrium (Proposition 9)](image)

Deriving the equilibrium outcomes for the other possible preference configurations is straightforward. Rather than state the propositions formally, the equilibria are simply listed in table 1.

Figure 5 graphs the policy outcomes for any pair of the candidates’ ideal policies when the vote share threshold equals 20%. The figure is somewhat ‘messier’ than the corresponding figures (3) for the policy outcomes under FPTP with and without (which is also equivalent to the outcomes for the top two...
**Table 1:** Equilibrium Strategies and Policy Outcomes Under the Majority Runoff with a Vote Share Threshold

(as $\varepsilon \to 0$)

<table>
<thead>
<tr>
<th>$x^1$</th>
<th>$x^2$</th>
<th>$p_1^*$</th>
<th>$p_2^*$</th>
<th>$p$</th>
<th>if</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; \frac{1}{2}$</td>
<td>$&gt; \frac{1}{2}$</td>
<td>$2\tau$</td>
<td>$1 - 2\tau$</td>
<td>$1 - 2\tau$</td>
<td>$x^2 \geq 1 - 2\tau$</td>
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</tr>
<tr>
<td>$&lt; \frac{1}{2}$</td>
<td>$&lt; \frac{1}{2}$</td>
<td>$[0, x^2)$</td>
<td>$x^2$</td>
<td>$x^2$</td>
<td>$x^2 &lt; 1 - 2\tau$</td>
</tr>
<tr>
<td>$&gt; \frac{1}{2}$</td>
<td>$&lt; \frac{2}{3}$</td>
<td>$[0, x^2)$</td>
<td>$x^2$</td>
<td>$x^2$</td>
<td>$2\tau &lt; 2x^1 - \frac{2}{3}$</td>
</tr>
<tr>
<td>$&gt; \frac{1}{2}$</td>
<td>$&gt; \frac{2}{3}$</td>
<td>$[0, \frac{1}{3}) \cup (2\tau, x^2)$</td>
<td>$\frac{4}{3}$</td>
<td>$\frac{2}{3}$</td>
<td>$2\tau &lt; 2x^1 - \frac{2}{3}$</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>$[0, 1 - x^2) \cup (2\tau, x^2)$</td>
<td>$2x^1 - 2\tau$</td>
<td>$2x^1 - 2\tau$</td>
<td>$2\tau \geq 2x^1 - \frac{2}{3}$</td>
</tr>
<tr>
<td>$&gt; \frac{2}{3}$</td>
<td>$&gt; \frac{2}{3}$</td>
<td>$x^1$</td>
<td>$[0, 1 - x^1) \cup (x^1, 1]$</td>
<td>$x^1$</td>
<td>$x^1$</td>
</tr>
</tbody>
</table>

runoff) an expressive candidate. Comparing the figures, darker shades indicate policies further to the right, suggests that policy outcomes under the majority run of with a vote threshold are more similar to the outcome under FPTP with an expressive candidate even when the threshold is fairly high. This stands to reason. The presence of an expressive candidate, and the possibility that she may advance onto the second ballot, ensures that the parties must take account of the candidate’s presence. As before the response is to adopt policy platforms further away from expressive candidates.

### 4 Discussion

The formal model presented here highlights in clear manner what may seem counterintuitive at first blush — the presence of expressive parties with extreme policy preferences results in policy moving away from the expressive party’s platform. Thus, while the rise of extremist parties is often seen as a worrisome development, the results here suggest that they actually serve to undermine the agenda of those parties. That is not to say that the extremist parties are harmless. In their absence, the expectation is that the policy-seeking parties will converge on the ideal policy of the median voter. In their presence, they adopt a policy platforms away from the median voter’s preferred policy. Their presence thus negatively affects the median correspondence (Powell & Vanberg, 2000; Golder & Stramski, 2010), the ideological proximity of the median voter and the median legislator, which can be considered a rough measure of voter
The effects of expressive parties on the strategies of policy-seeking, or main-
stream, parties have perhaps not attracted the attention they should in the
literature. As noted above, there is a large literature focused on the emergence
and success of extremist parties. These have included factors such policy po-
sitions of existing parties (see, e.g., Kitschelt & McGann, 1995; Van der Brug
et al., 2005), electoral institutions and other institutional barriers (see, e.g.,
Carter, 2002; Givens, 2005), and the internal organization of the extremist par-
ties (Kitschelt, 2007). With respect to the first of these, the model explored
here makes it clear that these questions cannot be answered adequately without
reference to the strategies adopted by existing parties. In particular, it is highly
questionable to attempt to explain the success of expressive parties on the basis
of the policy platforms adopted by the other parties. As we have seen, the
policy-seeking parties respond to the presence of an expressive party even when
that party has virtually no chance of winning (see Lemma 1). In other words,
the policy platforms of the policy-seeking parties are endogenous.\footnote{It is more reasonable to treat the parties’ positions as exogenous to the expressive party when addressing the emergence of new parties although Osborne (2000) suggests that existing parties may adopt entry-deterring strategies.}

\textbf{Figure 5: Policy Outcomes: Majority Runoff with a Vote
Threshold}
(as $\varepsilon \to 0$ for $\varepsilon$-Nash equilibria)
This becomes especially clear when one considers how the parties’ strategies depend on the electoral systems employed. In first-past-the-post elections we saw that both the policy-seeking parties have a tendency to move away from the expressive parties whereas in elections under majority runoff elections there exist scenarios in which the presence of expressive parties has a polarizing effect, i.e., the more left leaning party has an incentive to move to the left in order to limit the expressive success in advancing onto the second ballot. Not only does this affect the polarization (or convergence) of the mainstream parties, it also affects how successful the electoral appeals of the expressive party are.\footnote{Most of the literature examining the effect of electoral systems on the emergence and success of extremist parties simply operationalizes electoral institutions in terms of district magnitude (see, e.g., Carter, 2005; Golder, 2003). While the model presented here does not speak to proportional representation systems, it clearly demonstrates that FPTP and majority runoff systems have very different effects even though both systems have district magnitude of one.}

The more general lesson here is that more effort should be devoted to theorizing about the strategic incentives facing the parties and how these incentives are reflected in the positioning of the parties in the ideological space.

Of course, this is not the first contribution to the literature that emphasizes importance of the strategic reactions of established parties. Meguid (2005, 2010), e.g., considers how mainstream parties act strategically in response to niche parties. In particular, she assumes mainstream parties can choose among dismissive, accommodative, and adversarial strategies. In other words, Meguid introduces considerations of issue salience and ownership into the spatial model in order to offer insights into the strategic choices of mainstream parties and their consequences for the success of niche parties. While the model considered here does not explicitly consider salience or issue ownership it is interesting to consider her case study of the France in the context of the theory presented here.\footnote{Conceptually it is possible to think of no reaction to the presence of an expressive party as constituting a dismissive strategy in the model presented here although it doesn’t fully capture the notion of issue salience.}

Meguid (2010) shows that RPR (a Gaullist right party) and the Socialists reacted differently to the success of the National Front. The Socialists adopted an adversarial strategy, emphasizing its opposition to the National Front’s anti-immigration platform. The RPR, on the other hand, choose an accommodating strategy, adopting some of the National Front’s rhetoric. Legislative elections in France take place under a majority runoff system with a vote threshold of 12.5% of registered voters. Interestingly, the model presented here (Proposition
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9, and Table 1, provide the relevant predictions) predicts precisely this behavior, i.e., one mainstream party is expected to adopt an accommodating strategy in order to prevent the expressive party from advancing onto the second ballot, allowing the other mainstream party freedom to adopt an adversarial position, moving away from the median voter. Why then did the National Front become as big as it did, winning up to 15% of the vote 1997? The short answer is that for all its success in winning votes, it was also spectacularly unsuccessful in winning legislative seats — the National Front has only won a total of two seats, under the majority runoff, since it began contesting legislative elections in 1973.\textsuperscript{17} Following the logic of the model presented here, the RPR did just enough to prevent the National Front from advancing onto the second ballot in most districts. Attempting to crush the National Front would be too costly for the RPR, i.e., centrist voters would be more likely to switch their votes to the Socialists, allowing them to take further advantage by adopting a more leftist platform.

The model also suggests an explanation for the success of the National Front in comparison to far right parties in other countries that employ single member districts. The use of a majority runoff with a vote-threshold in France has two consequences. First, the two round system allows voters to separate expressive concerns from instrumental concerns because the outcome of the election is rarely decided on the first ballot — this is why the French are sometimes said to vote with their heart in the first round but with their head on the second. The second reason, suggested by the model, is that the presence of the vote-threshold means that deviating in policy away from the median voter (and away from the extreme right party) is smaller than in FPTP elections. In other words, the policy cost of expressive voting is smaller in runoff elections. For these reasons one might expect far right parties to be less successful in countries that use FPTP, which would appear to be true for the U.K. and the United States. Canada proves to be an exception as the Reform Party of Canada was quite successful. However, it can be argued that the Reform Party was quite different in that it was more of a regional party and it was actually quite successful in winning seats in the legislature, i.e., it looks less like an expressive party than, e.g., the National Front or U.K.’s BNP.

Australia, another country that employs single member districts, uses a

\textsuperscript{17}When France briefly adopted a proportional representation system for the 1986 elections the National Front secured 35 seats.
preferential system, the alternative vote. The alternative vote is very similar to the majority runoff — it is also known as the instant runoff. Under the alternative vote there are multiple rounds of counting — candidates are eliminated until some candidate has a majority of the vote. The system is, therefore, most similar to the top-two majority runoff system. Above it was shown that the presence of an expressive party has no policy effects under the top-two majority runoff. Far right parties are, therefore, expected to do better under the alternative vote than in FPTP systems and majority runoff elections with vote thresholds. That expectation is partially fulfilled. The nationalist far right party One Nation enjoyed some electoral success — it won 8.4% of the vote in 1998 but the party’s fortunes declined sharply in the following elections. Thus, the far right did better under the alternative vote than under the FPTP systems but arguably worse under the majority runoff with vote thresholds. It is important to note that a number of other factors shape the support of extremist right parties and that these comparisons ought to be taken with a grain of salt. The point of this discussion is intended to highlight some of the model’s predictions and to gauge their plausibility rather than a proper test of the model.

While far right parties have attracted a fair amount of scholarly attention, little has been written on how electoral institutions affect the policy consequences of their successes. Wong (2011), examining the politics of immigration control, is one exception. Focusing on the number of deportations, Wong finds that countries employing proportional representation systems are more aggressive when it comes to deportations. While the model cannot speak to the differences between proportional representation systems and the single-member district systems considered here, it provides a way to interpret this result.18 As we have seen here, mainstream parties are likely to adopt more pro-immigration policies in response to far right parties campaigning on immigration issues. The results derived here depend on the parties incentive to converge on some policy position — and in the presence of an expressive party policy position no longer corresponds to the median voter. While a similar effect may be present under proportional representation it is likely to be much smaller as the parties don’t face the same incentive to converge on a particular policy position, i.e., under proportional representation no single voter is pivotal to the outcome of the

18 Note, however, that Wong (2011) attributes the effect to the fact that anti-immigration parties are more likely to win legislative representation under proportional representation due to the lower effective threshold of representation. A possible objection to that argument is that extremist parties tend to be marginalized when it comes to coalition building in legislatures.
election. In proportional representation systems there is a greater tendency for the parties to spread out across the policy spectrum in which case the presence of an expressive party is less likely to affect the positioning of all the parties.\textsuperscript{19} Mair (2009) also argues that consensus democracies, one of whose characteristics is proportional representation, are more likely to rely on expressive modes of campaigning, i.e., parties are more likely to appeal to voters on the basis of group membership or particular values that the party stands for. If that is the case, the parties may have less scope for responding to extremist parties. Moreover, centrist parties may have little incentive to change their position — while a move to the left may win them more votes among left leaning voters it also means that they are less likely to win votes among right leaning voters.

5 Conclusions

In this paper I have considered how the presence of a non-competitive, expressive candidate influences electoral competition among policy-seeking candidates. The main result is that the presence of such a candidate results in the winning candidate adopting a policy platform away from the location of the median voter and further away from the expressive candidate’s preferred policy. I also considered how the rules of the game influence candidate positioning by comparing FPTP systems with the two major types of majority runoff systems. The comparison shows that the shift in policy away from the median voter’s preferred policy depends on the electoral system. The top-two runoff is shown to be immune to the presence of an expressive candidate (if she is sufficiently extreme) while systems employing vote thresholds do mitigate the policy shift (in comparison with FPTP) but does not eliminate it.

The results also suggest that the type of electoral system also influences the success of extremist parties in a more complicated manner than the literature has suggested. While in FPTP systems the strategic responses of the mainstream parties allow the extremist party to win a larger share of the vote, in majority runoff elections one of the mainstream parties may adopt a policy platform as to limit the size of the extremist party. Type of electoral system similarly affects the convergence of the mainstream parties’ platforms. Convergence, albeit not

\textsuperscript{19}Meguid (2005) do, however, argue that non-proximate parties have an incentive to respond to niche parties because raising the salience of the issue may help them out electorally. van Spanje (2010) similarly argues that anti-immigration parties affect the positioning of parties beyond the moderate right.
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necessarily on the median voter’s preferred policy, occurs in FPTP systems and in the top-two majority runoff while the mainstream parties’ platforms don’t always converge in majority runoff elections with vote thresholds.

In sum, the model considered here takes some initial steps in formally modeling electoral competition in the presence of expressive, extremist parties. While the results of the model may seem counterintuitive at first, the model reveals an underlying logic that is quite simple and, even, intuitive. However, more work remains to be done on the policy consequences of extreme and/or extremist parties. Of particular interest are the policy consequences of such parties in proportional representation systems. Intuitively, one might expect similar effects but, as discussed above, there are reasons to believe that the indirect policy effects might be smaller in proportional representation elections. On the other hand, because of lower thresholds of representation, extremist parties may have greater opportunities to influence policy directly. Examining these issues remains on the research agenda. In light of how electoral systems condition the policy consequences of electoral systems, examining them empirically is another avenue of research that deserves further exploration.
A Strategic Voting

We have assumed sincere voting in deriving our results in the body of the paper but there is a substantial literature that suggests that at least some voters cast their votes strategically. In this section we show that, under reasonable assumptions, our results do not depend on voters behaving sincerely. As generally is the case in models of multi-candidate elections multiple equilibria exist in the game even when weakly dominated strategies are eliminated. The reason is that in multi-candidate contests eliminating weakly dominated strategies only rules out the possibility that a voter votes for his least preferred candidate. In the current context, where a single extremist candidate runs against two ‘major’ candidates, it is, in a sense, reasonable to assume that (strategic) voters will coordinate on the major candidates.\(^{20}\) Fey (1997), e.g., shows how voters will coordinate on the leading candidates in the presence of opinion polls. Thus, if voters have prior expectations to the effect that the major candidates are the viable candidates it, in effect, becomes a self-fulfilling prophesy. Although assuming this particular form of strategic behavior, or coordination, is intuitively appealing it must be kept in my mind that other equilibria exist in the game.

I assume that a fraction \(\theta\) of voters are strategic and that both the strategic and sincere voters are uniformly distributed on the \([0, 1]\) interval. As in section 2.1, it is assumed that \(x^0 = 0\). Given platforms \(p_1\) and \(p_2\), and the assumption that the voters coordinate on the two major candidates, the candidates’ vote shares equal (if \(p_1 < p_2\)):

\[
\begin{align*}
v_0 &= (1 - \theta) \frac{p_1}{2} \\
v_1 &= \frac{p_2}{2} + \theta \frac{p_1}{2} \\
v_2 &= 1 - \frac{p_1 + p_2}{2}.
\end{align*}
\]

That is, supposing that candidates 1 and 2 are the front runners, voting for those candidates is the best response for the voters who rank either one of the candidates first. Candidate 0, on the other hand, only receives the votes of the sincere voters who most prefer the party. The remainder votes for the major candidate who adopts the more left-leaning platform. Thus, in general, only the

\(^{20}\) Note that this is actually not necessary for the case that we consider here as it is assumed that \(x^0 = 0\), in which case eliminating weakly dominated strategies is sufficient (i.e., voting for candidate 0 is weakly dominated for all voters as candidate 0 cannot win). It is only if \(x^0 > 0\) that making this additional assumption is helpful.
more left-leaning major candidate benefits from the voters' strategic behavior.

Under sincere voting, as shown in section 2.1, the platform $\frac{2}{3}$ replaced the median position as a center of attraction for the strategic candidates. In a similar manner, when a proportion, $\theta$, of the voters acts strategically the relevant platform equals $\frac{2}{3} + \theta$. It is easy to verify that the relevant platform is greater than $\frac{1}{2}$ for all $\theta < 1$, i.e., the policy outcomes diverges from the median position even when the tiniest proportion of voters acts sincerely. It is also evident that degree of divergence is strictly decreasing in $\theta$ with the policy outcome when $\theta = 0$ equaling $\frac{2}{3}$ and the median voter’s preferred policy obtaining when $\theta = 1$.

The characterization of the equilibria mirrors the characterization in section 2.1 with only the platform $\frac{2}{3} + \theta$ replacing the platform $\frac{2}{3}$. For sake of economy, I only consider the substantively most interesting case here (in which candidate 2 wins), i.e., where the candidates would locate at, or closer to, the median in the absence of an extremist candidate.

**Proposition 10 (Candidate 2 wins)** Suppose $x^1 \geq \frac{1}{3+\theta} + \frac{2}{3} \times \frac{2}{3}$ and $x^2 \geq \frac{2}{3} + \theta$. Any strategy profile $p^* = (0, p^*_1, p^*_2)$ where a) $p^*_1 \in [\frac{2}{3} + \theta - \varepsilon, \frac{2}{3} + \theta)$ and b) $p^*_2 \in W_2(p_1) \setminus (W_2(p_1) \cap W_2^{-}\varepsilon(p_1)) \cap (p^*_1, p^*_1 + \varepsilon)$ is a $\varepsilon$-Nash equilibrium of the game.

**Proof:** Candidate 0’s action is optimal as his utility only depends on his announced platform. Consider candidate 1’s action. The outcome of the election remains the same if candidate 1 deviates to $p^*_1 < p^*_1 - \varepsilon$. Condition b) and Lemma 1 imply that if candidate 1 deviates to $p^*_1 > p^*_1 + \varepsilon$ then candidate 2 remains the winner and the policy outcome is unchanged. Thus, candidate 1’s action is optimal. Candidate 2’s action is optimal because $x^2 > p^*_1$ and, by condition b), a deviation such that $p^*_2 > p^*_2 + \varepsilon$ results in candidate 1 winning the election and implementing $p^*_1$. As $u_2(p^*_1) < u_2(p^*_2)$, candidate 2’s action is optimal. $\square$

It is a simple matter to verify that the other propositions in section 2.1 can be replicated (with similar modifications) for when some voters act strategically. Thus, the presence of an extremist candidate with expressive motives leads the policy outcome to move away from the median voter, and away from the expressive candidate’s preferred policy, as long there are some sincere voters. The degree of divergence is, however, declining, in the number of strategic voters.
B  A Moderate Extremist

Above it was assumed that the extremist candidate was located on the fringe of the distribution of voter preferences. A consequence of this assumption is that the extremist candidate cannot win the election unless the other two candidates choose the same policy platform and that platform is further right than \( \frac{2}{3} \).

Assuming this degree of extremism is somewhat unrealistic but the result below shows that the same substantive conclusions are obtained if the expressive party is less extreme.

We retain the assumption that the expressive party is extremist and assume \( x^0 \in [0, \frac{1}{6}] \), i.e., no more than one-third of the voter population is more extreme (counting both left and right extremists) than the expressive candidate. It is a simple matter to characterize the equilibria in the presence of a centrist expressive party but our primary interest here is to examine the policy effects of extremist parties. I continue to assume that \( x^0 < x^1 < x^2 \).

Intuitively, as the expressive party becomes more moderate it attracts more votes, ceteris paribus, and the major parties must meet this challenge by adopting a more centrist position.

**Proposition 11 (Candidate 2 wins)** Suppose \( x^1 \geq \frac{1}{3} - \frac{x^0}{2} + \frac{x}{2} \) and \( x^2 \geq \frac{2}{3} - x^0 \). Any strategy profile \( p^* = (0, p^*_1, p^*_2) \) where a) \( p^*_1 \in (\frac{2}{3} - x^0 - \varepsilon, \frac{2}{3} - x^0) \) and b) \( p^*_2 \in W_2(p^*_1) \setminus (W_2(p^*_1) \cap W_2^{-\varepsilon}(p^*_1)) \cap (p^*_1, p^*_1 + \varepsilon) \) is a \( \varepsilon \)-Nash equilibrium of the game.

**Proof:** Candidate 0’s action is optimal as his utility only depends on his announced platform. Consider candidate 1’s action. The outcome of the election remains the same if candidate 1 deviates to \( p^*_1 < p^*_1 - \varepsilon \). Condition b) and Lemma 1 imply that if candidate 1 deviates to \( p^*_1 > p^*_1 + \varepsilon \) then candidate 2 remains the winner and the policy outcome is unchanged. Thus, candidate 1’s action is optimal. Candidate 2’s action is optimal because \( x^2 > p^*_1 \) and, by condition b), a deviation such that \( p^*_2 > p^*_2 + \varepsilon \) results in candidate 1 winning the election and implementing \( p^*_1 \). As \( u_2(p^*_1) < u_2(p^*_2) \), candidate 2’s action is optimal. \( \square \)
References


