Expressive Motives, Third-Party Candidates, & Voter Welfare

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Abstract

The motives of 'third-party' candidates in plurality and majority run-off elections are somewhat of a puzzle. Almost by definition, third-party candidates do not stand a chance of winning elections so their motives must derive simply from a desire to be in the limelight of politics or perhaps from a hope of drawing attention to particular issues in the event that it will be co-opted by one of the major candidates. A third possibility, explored here, is that the strategic actions of other candidates and voters lead to policy outcome favorable to the third-party candidate and his constituency. In the paper I explore whether there exist conditions under which this is the case. In plurality rule systems the opposite is a more likely scenario – the presence of a third-party candidate leads the major party candidates to position themselves away from the median voter and further away from the third-party candidate's constituency. Majority run-off systems, on the other hand, provide third-party candidates with a greater opportunity to run for office without adversely influencing (from their supporters' point of view) the outcome.

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1 Introduction

Extremist parties have received considerable attention in the literature. Much of the literature has been concerned with the rise of extremist parties and the conditions under which extremist parties appear and become successful. Less effort has been devoted to examining the consequences of extremist parties. Extremist parties, where they have been successful, have by-and-large been excluded from government participation. When they are allowed to take part, havoc normally ensues. The EU countries reacted harshly to the formation of a coalition between the Austrian Freedom Party and the People's Party in 2000 and threatened to boycott the government politically. In France, the cooperation between the centre-right and the National Front at the regional level was frowned upon by national party organization and eventually led to an electoral reform for the regional elections which aimed at creating large majorities to remove the temptation of the regional parties to look to the National Front for support.

If extremist parties are marginalized in the coalition formation and the policy making processes one might assume that extremist parties simply have no influence on policy. If that is the case the attention devoted to extremist in both journalistic and scholarly accounts would appear to be much ado about nothing. But it also raises some interesting questions. If it is really the case that extremist parties have no impact on policy why, then, do voters cast their votes for extremist parties and, perhaps more importantly, why do extremists politicians run for office? A simple answer might posit that extremist voters and candidates simply use election to express their preferences. A somewhat more sophisticated answer would consider the extremist votes as signals that influence future electoral outcomes Smirnov & Fowler (2007).

I do not answer these questions in this paper. Instead, I show that extremist participation is even more puzzling than is suggested above. The participation of extremist parties is not likely to move the policy outcome *closer* to the extremist party's preferred policy outcome but *further away* from it.

2 Extremist Parties & Policy

The electoral success of extremist parties invariably attracts considerable attention. As extremist parties rarely have the opportunity to make policy or even to influence it much, all the attention heaped upon extremist parties appears to unwarranted or have to do with factors besides political outcomes, e.g., concern about the prevalence of extremist views in society. But it is also possible that extremist parties affect policy outcomes despite the fact that they are marginalized. Whether this is the case has not been adequately addressed in the literature although it would be unfair to say that the issue has been ignored.

Some models of electoral competition in parliamentary systems implicitly allow extremist parties to influence policy outcomes. These models assume that the implemented policy outcome equals the weighted average of the parties' vote shares.[1] Thus, the support of extremist parties has a direct impact on policy outcomes, providing both extremist candidates to run and extremist voters to vote for them. Unfortunately, as De Sinopoli & Iannantuoni (2001) show, it is not only rational for extremist voters to vote for extremist candidates, it is also rational for centrist voters to voter for extremist parties. By doing so, the centrist voters can edge the policy outcome slightly to the left or the right – the only centrist voters that don't have an incentive to vote for extremist parties are those who get their most preferred policy implemented. In equilibrium then, only the two most extreme parties receive votes although the policy outcome will be centrist. Kedar (2005) argues that this logic accounts for the tendency of voters to vote for parties that have more extremist preferences than they do themselves. In her view, these incentives are, however, tempered, by expressive preferences, i.e., that voters don't only care about policy outcome but also care about voting for parties that they find agreeable.

The problem with the above models is that they abstract away one of the most crucial feature of parliamentary systems. Legislatures are by-and-large majoritarian creatures and legislative powers are largely invested in the hands of a government that has the support, explicit or implicit, of a majority of the legislature. In other words, the legislative power of opposition parties in parliamentary systems is generally negligible.

[1] See, e.g., Kedar (2005) and Ortuño-Ortín (1997).

3 Competing with Extremist under FPTP

We start by considering a simple model of electoral competition between a party driven by expressive motives and two parties who have preferences over government policy. The expressive party derives its utility from simply taking a stand, i.e., running for office on its most preferred platform. If the expressive party's most preferred platform is extreme we can think of the party as an extremist party that contests the election without regard to its potential to influence policy outcomes. The other two parties may have extreme policy preferences but what distinguishes them from the expressive party is that they care about the implemented policy outcome rather than simply taking a stand. Although we do not rule out the possibility that the policy seeking parties have extreme preferences it is somewhat natural to think of this scenario as an electoral contest between an extremist party and two moderate parties.

Formally, let the set of parties be $\{0, 1, 2\}$. The set of outcomes of the game is $[0, 1]$ and each parties' most preferred policy outcome is denoted x^k. Without loss of generality it is assumed party 0 has expressive preferences and that $x^0 \leq x^1 \leq x^2$. The set of preference profiles is assumed to be single-peaked on $[0, 1]$. That is, for expressive candidates, the payoff is decreasing in the distance between their ideal policy and their announced platform and, for non-expressive candidates, each candidate's payoff is decreasing in the distance between her ideal policy and the implemented policy.

Voters' preferences over policy are defined in an analogous manner to the candidate preferences and the distribution of the voters' of ideal policies is described by the cumulative distribution function $F(x)$. We suppose throughout that $f(x)$ is uniform and $x^0 = 0$.

The game consists of two stages. At the first stage, each candidate chooses a policy platform, $p_k \in [0, 1]$. At the second stage, the voters cast a vote for one of the candidates, $a_k \in \{0, 1, 2\}$. The candidate that receives the most votes wins the election and implements her policy platform. That is, it is assumed that candidates can credibly commit to their policy platform. Voters are assumed to cast their votes sincerely, i.e., $a_i = k$ only if $u_i(p_k) \geq u_i(p_j), \forall j \neq k$.

As it turns out, a pure strategy Nash equilibrium often doesn't exist in the game. As a solution concept we therefore consider a slightly modified version of a Nash Equilibrium in which the candidates don't consider very small deviations from their strategies. That is, given a strategy profile in which candidate k chooses platform p^k then the candidate will not consider deviations that involve the strategies in $[p^k - \epsilon, p^k + \epsilon]$. We term this equilibrium concept ϵ-Nash equilibrium. This requirement is fairly weak because ϵ can be arbitrarily small.[2]

For sake of comparison it is instructive to consider the outcome of the game in the absence of an expressive candidate. This result is familiar from the literature (see, e.g., Osborne, 1995). The median voter plays an important role in the competition between policy motivated candidates much as is the case with office-seeking candidates. In short, the median voter determines the policy outcome in a simple manner. If the two candidates' ideal policies are located at opposite sides of the median voter the equilibrium policy in the game corresponds to the median voters preferred policy. If the two candidates' ideal policies are located at the same side of the median voter the equilibrium policy corresponds to the ideal policy of the candidate that is closer to the ideal policy of the median voter. It is a simple matter to verify that the ε-Nash equilibrium policy outcomes, in the limit as ε tends to zero, will be identical.

The presence of an extremist candidate changes the dynamics of the competition somewhat. In particular, it is no longer true that the median voter holds the privileged position described above. Instead, if the expressive candidate's most preferred policy is $x^0 = 0$, the voters located at $\frac{2}{3}$ will often be in a pivotal position.

Before we derive the equilibria of the game it is useful to provide some additional notation. We use p_k^* to denote candidate k's equilibrium strategy. For any policy p_k, define $\bar{p}_k$ and $\underline{p}_k$ such that $\left|\frac{2}{3} - p_k\right| = \left|\frac{2}{3} - \bar{p}_k\right| = \left|\frac{2}{3} - \underline{p}_k\right|$ and $\bar{p}_k < \underline{p}_k$. Thus, in those cases where the votes of the voters with ideal policies located at $\frac{2}{3}$ are pivotal, candidate j can beat candidate k by choosing a platform in $W_j(p_k) = (\underline{p}_k, \bar{p}_k)$.

It is convenient to additionally define the sets of positions that candidate j, whose platform is located in the winset of p^k, can reach by the smallest possible change (ε) in her platform. We let $W^{+\varepsilon}(p_k)$ denote the set of such positions that candidate j can reach by moving to the right and $W^{-\varepsilon}(p_k)$ denote the set of such positions that candidate j can reach by moving to the left.

[2]Another way to solve the equilibrium non-existence problem would be to simply assume that the parties must differentiate themselves by not adopting platforms that are too similar. The approach used here is similar to an ϵ-equilibrium in which players ignore deviations that increase their payoff by less than ϵ (Radner, 1980).

Thus, $W_j^{+\varepsilon}(p_k) = (\underline{p}_k + \varepsilon, \bar{p}_k + \varepsilon)$ and $W_j^{-\varepsilon}(p_k) = (\underline{p}_k - \varepsilon, \bar{p}_k - \varepsilon)$.

The intersection $W_2(p_1) \cap W_2^{-\varepsilon}(p_1)$ then represents the platforms $p_2 \in W_2(p_1)$ such that there exist a deviation $p_2' > p_2 + \varepsilon$ which is still winning. Consequently any platform $p_2 \in W_2(p_1) \cap W_2^{\varepsilon}(p_1)$ cannot be an equilibrium strategy if candidate 2 prefers a policy further to the right. That is, if $p_2 \in W_2(p_1) \cap W_2^{\varepsilon}(p_1)$ there exists another policy in the winset that candidate 2 prefers more. On the other hand, any $p_2 \in W_2(p_1) \setminus (W_2(p_1) \cap W_2^{-\varepsilon}(p_1))$ offers no opportunities for profitable deviations. Similarly, $p_1 \in W_1(p_2) \setminus (W_1(p_2) \cap W_1^{+\varepsilon}(p_2))$ represent the platforms that candidate 1 cannot deviate from to a platform further to the left without losing the election.

We start out by considering what the prospects of winning for the expressive candidate are.

Lemma 1 *Candidate 0 loses with certainty if* $p_1 \neq p_2$.

Proof: Suppose $p_1 < p_2$. Then candidate 0's vote share, v_0, equals $\frac{p_1}{2}$. The vote shares of the other candidates are $v_1 = \frac{p_1+p_2}{2} - \frac{p_1}{2} = \frac{p_2}{2}$ and $v_2 = 1 - \frac{p_1+p_2}{2}$. Thus, $v_1 > v_0$ by $p_1 < p_2$. □

Lemma 1 shows that the expressive candidate never wins as long as the other two candidates don't adopt the same platform. It is a simple matter to verify that the expressive candidate can win if the other candidates do adopt the same platform. Suppose for example that candidates 1 and 2 locate at 1. In this case candidate 0 receives half the vote, while candidates 1 and 2 split the other half between them. Thus, unless the voters are able to coordinate their actions candidate 0 wins.

Lemma 1 also shows that the extremist party in this model is a marginal party in the sense that its hopes of winning the elections are, a priori, almost non-existent. The assumptions we make in our model can be considered strict in this sense but as we shall see, this does not imply that such parties are irrelevant for the outcome of the elections.

It is worthwhile briefly considering how relaxing our assumptions might alter Lemma 1. Assuming a uniform distribution of voters is arguable a strict assumption. However, Lemma 1 holds for any distribution that is symmetric about .5 with $f'(x) > 0, \forall x < .5$. The assumption that $x^0 = 0$ is more crucial here. More action profiles result in candidate 0 winning if x^0 increases. Thus, our results here apply perhaps most clearly to expressive extremist parties.

The effects of less extremism are, however, a matter of degree and don't result in qualitatively different results. Somewhat counterintuitively, less extremism leads to more moderate positions by the other parties.[3]

Proposition 1 (Candidate 2 wins) *Suppose* $x^1 \geq \frac{1}{3} + \frac{\varepsilon}{2}$ *and* $x^2 \geq \frac{2}{3}$*. Any strategy profile* $p^* = (0, p_1^*, p_2^*)$ *where a)* $p_1^* \in [\frac{2}{3} - \varepsilon, \frac{2}{3})$ *and b)* $p_2^* \in W_2(p_1) \backslash (W_2(p_1) \cap W_2^{-\varepsilon}(p_1)) \cap (p_1^*, p_1^* + \varepsilon)$ *is a* ε*-Nash equilibrium of the game.*

NOTE: ALSO NECESSARY CONDITIONS FOR CANDIDATE 2 WINNING

Proof: Candidate 0's preferences only depend on his announced platform. Choosing x^0 is therefore optimal. Now consider candidate 1's strategy. Deviation to any $p_1' < p_1^* - \varepsilon$ has no effect on the outcome. Candidate 1's vote share remains unaltered and candidate 2 wins and implements p_2^*. By b) and Lemma 1 a deviation to $p_1' > p_1^* + \varepsilon$ has no effect on the outcome, i.e., candidate 2 still wins. Candidate 1's strategy is thus optimal. As candidate 2 wins under the strategy profile p^*, the only question is whether candidate 2 can obtain a more favorable outcome. Since $p_1^* < p_2^*$ we only need to consider deviations such that $p_2' > p_2^* + \varepsilon$. By b), $p_2' \notin W_2(p_1^*)$ and any deviation $p_2' > p_2^* + \varepsilon$ would result in candidate 1 winning the election and implementing p_1^*. Candidate 2's strategy, p_2^* is optimal as $u_2(p_2^*) > u_2(p_1^*)$. □

Figure 1 depicts the logic of proposition 1. In the figure candidate 2 wins on platform p_2^*. Candidate 1's action is optimal as deviating to any $p_1' < p_1^*$ doesn't change the outcome of the election. While there exist platforms that lead to candidate 1's victory, e.g., $p_1' = \frac{2}{3}$, these platforms are not obtainable as candidates only consider deviations that are larger than ε. Thus, any possible deviation $p_1' > p_1^*$ would require adopting a platform greater than p_2^* in which case candidate 2 remains the winner. Similarly there exist winning platforms that candidate 2 would prefer to p_2^* but the restriction to consider only changes greater than ε render those unattainable.

Thus, it should be clear that the equilibria described in proposition 1 are not Nash equilibria. This, of course, begs the question of how reasonable the restriction to ε-Nash equilibria is. We think this restriction is reasonable for two reasons. First, for the parameters of game in proposition 1, there exist no Nash equilibria. However, the players' best responses tell us very clearly what the players' incentives are. That is, each player has a strong incentive to locate closer to $\frac{2}{3}$ than their non-expressive competitor. Yet, locating at $\frac{2}{3}$ is not an equilibrium strategy because if both non-expressive candidates

[3] These results are not included in the current version of the paper.

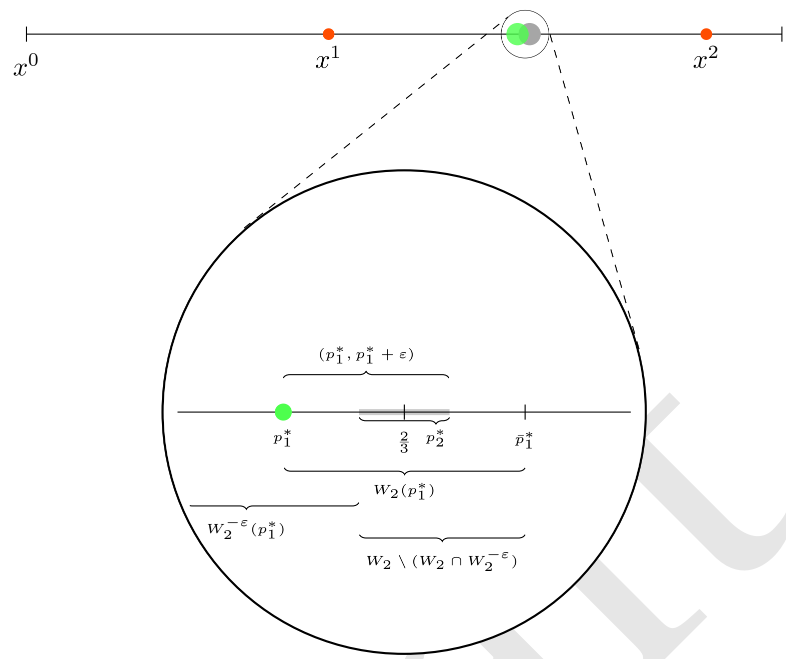

FIGURE 1: EXAMPLE OF AN EQUILIBRIUM (PROPOSITION 1)

locate at $\frac{2}{3}$ then the three candidates tie and the expected payoff to each of the non-expressive candidates is lower than if they concede the election to the other non-expressive voter. However, by doing so, and choosing some other platform, it opens up profitable deviations for the candidate located at $\frac{2}{3}$. In other words, the platform $\frac{2}{3}$ acts as a strong center of gravity but the continuity of the policy space rules out the existence of a Nash equilibrium. Intuitively, however, one would expect the parties to locate close to $\frac{2}{3}$.

Second, as we mention above, the minimum deviation ε can be made arbitrarily small, which suggests that in substantive terms the assumption is not very strong. Thus, instead of ε corresponding to changing a section in a party's manifesto it can correspond to a choice of a single adjective. In other words, we don't find it implausible to assume that parties ignore making such small changes (or alternatively, that they will be lost on voters).

Proposition 1 demonstrated the gravitational force of the platform $\frac{2}{3}$. The equilibrium policy outcome will be in the neighborhood of $\frac{2}{3}$ and equals $\frac{2}{3}$ in the limit as ε tends to zero. Under proposition 1 candidate 2 wins the election with certainty. However, there also exist equilibria in which candidate 1 wins the election with certainty on a platform in the neighborhood of $\frac{2}{3}$. Proposition 2 characterizes these equilibria.

Previously we defined $W_j(p_k)$ as the winset for candidate j given a candidate k's policy platform. That is, $W_j(p_k) = (\underline{p}_k, \bar{p}_k)$. It is convenient to additionally define to consider the 'shifted' winset $W_j^{+\varepsilon}(p_k) = (\underline{p}_k + \varepsilon, \bar{p}_k + \varepsilon)$ for candidate 1. The intersection $W_1(p_2) \cap W_1^{+\varepsilon}(p_2)$ then represents platforms the platforms in $W_1(p_2)$ such that there exist a deviation $p_1' < p_1 - \varepsilon$ which is still winning. Consequently any platform $p_1 \in W_1(p_2) \cap W_1^{\varepsilon}(p_2)$ cannot be an equilibrium strategy. On the other hand, any $p_1 \in W_1(p_2) \setminus (W_1(p_2) \cap W_1^{+\varepsilon}(p_2))$ offers no opportunities for profitable deviations.

Proposition 2 (Candidate 1 wins) *Suppose $x^1 \geq \frac{1}{3} + \frac{\varepsilon}{2}$ and $x^2 \geq \frac{2}{3}$. Any strategy profile $p^* = (0, p_1^*, p_2^*)$ where a) $p_2^* \in [\frac{2}{3} + \varepsilon, \frac{2}{3})$ and b) $p_1^* \in W_1(p_2^*) \setminus (W_1(p_2^*) \cap W_1^{\varepsilon}(p_2^*)) \cap (p_2^* - \varepsilon, p_2^*)$ is a ε-Nash equilibrium of the game.*

NOTE: ALSO NECESSARY CONDITIONS FOR CANDIDATE 1 WINNING

Proof: Candidate 0's preferences only depend on his announced platform. Choosing x^0 is therefore optimal. Now consider candidate 1's strategy. Candidate 1 wins the election and, by $p_1^* \in W_1(p_2^*) \setminus (W_1(p_2^*) \cap W_1^{\varepsilon}(p_2^*))$ there exists no platform such that candidate 1 prefers and she still wins if she adopts it. Candidate 1 doesn't prefer any deviation that results in candidate 2 winning by $x^1 \leq p_1^* < p_2^*$. Thus, candidate 1's strategy is optimal. Any deviation $p_2' > p_2^*$ only reduces candidate 2's vote share. A deviation $p_2' < p_2^*$ may result in candidate 2's victory but, by $p_1^* \in (p_2^* - \varepsilon, p_2^*)$, the deviation will be further away for candidate 2's ideal policy than candidate 1's platform, i.e., $p_2' < p_1^*$. Therefore, no profitable deviations exist for candidate 2. □

The logic behind proposition 2 is analogous to proposition 1. The only difference is that candidate 1 wins. The policy outcome in the limit as ε goes to zero remains $\frac{2}{3}$. So far we have assumed that the policy preferences of the two policy motivated candidates are such that one of the parties prefers a policy relatively far to the right ($x^2 > \frac{2}{3}$) while the other is centrist ($\frac{1}{3} < x^1 < \frac{2}{3}$). We now consider the case when candidate 2's ideal policy is less than $\frac{2}{3}$. In this scenario candidate 1's position is in a certain sense weaker than in the scenario considered before because candidate 1 will lose if he adopts a platform anywhere between 0 and x^2 – provided that candidate 2's platform doesn't lie to the right of $\frac{2}{3}$. Moreover, candidate 1 will rarely gain from choosing a platform to the right of x^2. The reason the candidate might want to do so is if he preferred the platform 0 to x^2. However, because $x^2 < \frac{2}{3}$ choosing such a platform is no longer sufficient to make candidate 0 the winner (again, if $p_2 < \frac{2}{3}$). Because the ideal policy of the pivotal voter is located in a subset of the policy space $(x^2, 1)$ that contains neither of the candidates' ideal platforms a Nash equilibrium will exist. Proposition 3, therefore, characterizes the Nash equilibria of the game. It is easily verified that the ε-Nash equilibria will result in a very similar policy outcome.

Proposition 3 *Suppose $x^2 \leq \frac{2}{3}$. Any strategy profile $p^* = (0, p_1^*, x^2)$ where $p_1^* \in [0, x^2) \cup (1 - x^2, 1]$ is a Nash equilibrium of the game.*

Proof: Candidate 0's strategy is optimal as x^0 is the platform that maximizes her utility. Similarly, candidate 2's is optimal as the outcome of game when the players take the actions described in the proposition equals x^2. Finally, p_1^* is optimal because only if candidate 1 deviates to $p_1' \in (x^2, 1 - x^2)$ does candidate 1 win. By $x^1 < x^2$, candidate 1 will be made worse of by any such deviation. □

Figure 2 graphs the possible equilibria of the game. Candidate 2 always locates at x^2 and wins while candidate 1 locates either to the left of candidate 2 or sufficiently far to the right to ensure that candidate 2 wins the election.

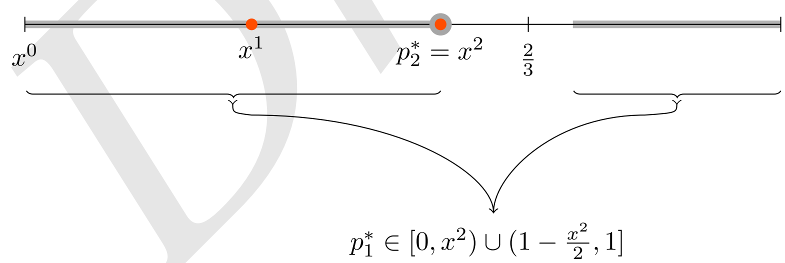

FIGURE 2: EXAMPLE OF AN EQUILIBRIUM (PROPOSITION 2)

Finally, we characterize the equilibrium when candidate 1's ideal platform is in the first third of the policy space and candidate 2's ideal platform is in the last third of the policy space. As in propositions 1 and 2 the only possible equilibrium policy outcome is in the neighborhood of $\frac{2}{3}$.

Proposition 4 *Suppose $x^1 < \frac{1}{3}$ and $x^2 \geq \frac{2}{3}$. Any strategy profile $p^* = (0, p_1^*, p_2^*)$ such that either:*

a) $p_1^* \in [\frac{2}{3} - \frac{\varepsilon}{2}, p_2^*)$ *and*

b) $p_2^* = (\frac{2}{3} - \frac{\varepsilon}{2}, \frac{2}{3})$

or

c) $p_1^* \in (p_2^* - \varepsilon, \frac{2}{3})$ *and*

d) $p_2^* = [\frac{2}{3}, \frac{2}{3} + \varepsilon)$

and is a ε*-Nash equilibrium of the game.*

Proof: Candidate 0 maximizes its possible payoff by choosing the platform x^0. Suppose conditions a) and b) hold. A deviation to $p_1' < p_1^* - \varepsilon$ by candidate 1 doesn't change the outcome of the election. That is, candidate 2 will still receive more than $\frac{1}{3}$ of the vote and win the election. A deviation $p_1' > p_1^* + \varepsilon$ only results in winning if $p_1' > p_2^*$ but then $u_1(p_2^*) > u_1(p_1')$. Candidate 2 wins and therefore any deviation $p_2' < p_2^*$ is not profitable. By condition a) there exist no deviations $p_2' > p_2^*$ such that candidate 2 remains the winner. Now suppose conditions c) and d) hold. By condition c), there exists no deviation for candidate 1 that results in him winning the election. Candidate 1's strategy is therefore optimal. Candidate 2 wins and, therefore, only deviations such that $p_2' > p^*$ need to considered. For any such deviation $\frac{2}{3} - p_1^* < p_2' - \frac{2}{3}$, by condition c) and d), in which case candidate 1 wins and implements p^*1. As $u_1(p_2^*) > u_1(p_1^*)$, candidate 2's strategy is optimal. □

Together propositions 1-4 characterize the equilibria of the game. It is instructive to compare the equilibrium outcomes in the presence of an expressive candidate with the standard model of electoral competition between policy-seeking candidates. Figure **??** graphs the policy outcome as a function of the two policy-seeking candidates' ideal policies. The presence of an expressive candidate has clear implications for policy. The policy outcome with an expressive candidate, p^e, is always further to the right as the policy outcome without an expressive candidate, p^s, i.e., $p^e > p^s$, as long as the two policy-seeking candidates' ideal policies aren't too far to the right ($x^1, x^2 > \frac{2}{3}$).

Thus, the consequence of an expressive (and extreme) party entering the context has the effect of moving the policy further away from that party's ideal policy outcome. It should be clear that if the party were assumed to care about policy and to make a strategic decision whether to enter the race then not entering would lead to a better outcome than adopting an extreme

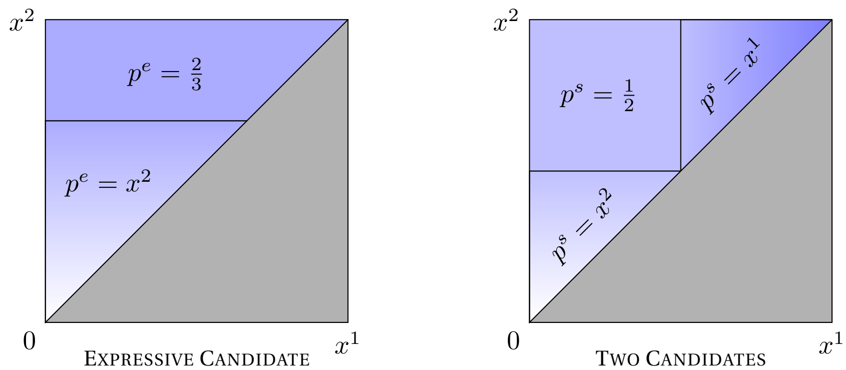

FIGURE 3: POLICY OUTCOMES WITH & WITHOUT AN EXPRESSIVE CANDIDATE (AS $\varepsilon \to 0$ FOR ε-NASH EQUILIBRIA)

platform. The assumption of expressive position taking is arguably crucial to obtain the results above but they nevertheless highlight the policy consequences of non-competitive, or 'non-coalitionable', parties. In particular, it raises the question of why such parties choose to contest elections.

It may also raises the question of how sensitive the results above are to the specific assumptions made above. There are three assumptions that may look particularly important. First, the expressive candidate is assumed to be *extremely* extreme, i.e., her ideal policy is at the far left end of the voters' ideal point distribution. Allowing the candidate less extreme policy preferences would not change the substantive finding but the rightward policy shift would be smaller. Second, the distribution of voters' ideal points is assumed to be uniform. Again, substantive the results remain the same for most unimodal distributions but, again, the effect of an expressive candidate would tend to be smaller. Third, the candidates are assumed to be purely policy-seeking. Office motives have less effect on the policy outcome – the equilibrium policy outcome corresponding to the scenarios in propositions 1 and 2 remains $\frac{2}{3}$ (assuming candidates cannot pick the same platform). The effect of office motives in combination with policy motives would change the equilibrium conditions but would not, in general, have much of an impact on the equilibrium policy outcome.

The main result is, therefore, likely to survive weakening of the assumptions about candidate location, voter distribution, and politicians' motives (though deriving the equilibria becomes a bit messier). The question why extreme candidates run, given their 'adverse effects' on policy, thus remains unanswered. One factor, frequently cited in the literature as determining the success of extremist parties, is the type of electoral systems in use. While the effects of different types of electoral system is beyond the scope of this paper we now adopt our model to majority run-off systems.[4]

4 Competing with Extremists: The Majority Runoff

The majority runoff is a class of electoral systems although it often is identified with its most common variant in which the top two candidate on the first round of contest advance onto the second, and final, ballot. However, majority runoff systems can vary along three dimensions: The number of ballots, the threshold for advancing onto a subsequent ballot (defined in terms of rank or vote share), and the winning requirements for each ballot (Indridason, 2007). Below we consider the two most prominent variants of the majority runoff, a top-two runoff (described above) and a two ballot runoff with a vote share threshold. Well-known examples of these electoral systems are the French presidential (top-two runoff) and legislative elections (vote share threshold).

The model in the previous section only needs slight modification to accommodate runoff elections. First, there are now two rounds of voting. We assume that the voters cast their votes sincerely on both ballots. Second, incorporating the threshold for inclusion on the second ballot is straightforward. Third, we assume that the candidates stand on the same platform in both the first and the second round of the election. The players' action sets and payoffs are defined in the same way as before.

4.1 The Top-two Runoff

In the top-two runoff, the two candidates receiving the most votes on the first ballot advance onto the second ballot where the candidate receiving a majority of the vote is declared the winner. The strategic situation of the policy-seeking candidates under the (top-two) runoff differs from their situation un-

[4]Extending the model to proportional systems is not a trivial matter. Equilibria often fail to exist in candidate position games under proportional representation.

der FPTP because, first, they may care about which candidates advance onto the second ballot and, second, the absence of a third candidate on the second ballot influences the identity of the pivotal voter. In general, the effect is that candidates tend to choose more moderate position.

The equilibrium policy outcome depends on the policy preferences of the two policy-seeking candidates in a rather predictable manner. The equilibrium of the game is characterized in the following three propositions.

Proposition 5 *Suppose* $x^1 \geq \frac{1}{2}$ *and* $x^2 \geq \frac{1}{2}$. *Any strategy profile* $p^* = (0, x^1, p_2^*)$ *such that* $p_2^* \in (p_1^*, 1] \cup [0, 1 - p_1^*]$ *is a Nash equilibrium of the game. The equilibrium policy outcome equals* x^1.

NOTE: CANDIDATE 0'S VOTE EQUALS $\frac{x^1}{2} < 0.5$ BY $x^1 < x^2 \leq 1$

Proof: Candidate 0's strategy is optimal as she receives her maximal payoff. Candidate 1's strategy is optimal for the same reason. For candidate 2, any deviation such that $1 - p_1^* < p_2' < p_1^*$ will result in candidate 2 advancing onto the second ballot and subsequently winning the second ballot. Then the policy outcome is p_2' but $u_2(p_2') < u_2(p_1^*)$. Hence, such deviation is not profitable. □

Proposition 6 *Suppose* $x^1 < \frac{1}{2}$ *and* $x^2 > \frac{1}{2}$. *The strategy profiles* $p^* = (0, \frac{1}{2}, p_2^*)$ *and* $p^* = (0, p_1^*, \frac{1}{2})$ *where* $p_k^* \in [0, 1], k \in \{1, 2\}$ *are Nash equilibria of the game. The equilibrium policy outcome equals* $\frac{1}{2}$.

Proof: Candidate 0's strategy is optimal as she receives her maximal payoff. Suppose candidate k locates at $\frac{1}{2}$. Then candidate k wins the election independent of the action of candidate $j \in \{1, 2\} \setminus k$. If $p'j < \frac{1}{2}$ candidate k wins more than half the vote and advances onto the second ballot (or alternatively, is declared the winner right away) where she wins the election. If $p_j' > \frac{1}{2}$ candidate k wins more than a quarter of the vote while candidate 0 wins exactly a quarter. Candidate k thus advances and wins a majority on the second ballot. If $p_j' = \frac{1}{2}$, candidates 1 and two win $\frac{3}{8}$ of the vote each and advance onto the second ballot. In each case the policy outcome equals $\frac{1}{2}$ and thus all the candidates' strategies are optimal. □

Proposition 7 *Suppose* $x^1 \leq \frac{1}{2}$ *and* $x^2 \leq \frac{1}{2}$. *Any strategy profile* $p^* = (0, p_1^*, x^2)$ *such that* $p_1^* \in [0, p_2^*] \cup (1 - p_2^*, 1]$ *is a Nash equilibrium of the game. The equilibrium policy outcome equals* x^2.

Proof: Candidate 0's strategy is optimal as she receives her maximal payoff. Candidate 2's strategy is optimal for the same reason. For candidate 1, any deviation such that $p_2^* < p_1' < 1 - p_2^*$ will result in candidate 1 advancing onto the second ballot and subsequently winning the second ballot. Then the policy outcome is p_1' but $u_1(p_1') < u_1(p_2^*)$. Hence, such deviation is not profitable. □

The equilibria of the game should look rather familiar from our discussion of the standard model of electoral competition between two policy seeking candidates. That is, under the majority runoff the presence of an extremist candidate has no effect on the strategies pursued by the policy seeking candidates and the game is equivalent to the standard two candidate model. This is not too surprising – when the expressive party advances onto the second ballot it is always pitted against a more moderate party and is therefore sure to lose. As such the result is in some sense rather trivial.

However, we should like to point out, this corresponds to the case that we were interested in exploring, i.e., the case of non-competitive or 'non-coalitionable' parties. The results show in a very stark manner what the effects of the majority runoff are, i.e., they neutralize non-competitive parties. It also suggests that the relationship between electoral systems and successful extremist parties is more complex than the literature suggests. That is, it may not simply be a matter of electoral systems differing in terms of their thresholds of representation, which in turn provides extremist candidates with an incentive to run and extremist voters to vote for them. Our results suggest that these decisions should also be influenced by how the decision to run influences the strategies of other candidates. Extremist candidates may be less willing to contest elections, and extremist voters may be less willing to vote for extremist candidates, if the consequence is that the policy outcome becomes less favorable to them.

Meguid (2005), e.g., does consider how mainstream parties might act strategically in response to niche parties. In particular, she assumes mainstream parties can choose among dismissive, accommodative, and adversarial strategies and considers how these different strategies might affect their support. The results here suggest that the type of electoral system will determine the strategic response of mainstream parties. Thus, in Meguid's terminology, mainstream parties in FPTP systems are found to adopt adversarial strategies while under the majority runoff they are dismissive. However, although not modeled here, when the parties' strategies are put in the context of likely effects on policy outcomes the predicted effects of the strategies on the success of niche parties are the opposite of what Meguid argues they should be. Meguid argues, e.g., that the adoption of dismissive strategies should lead to the niche party losing votes. The exact opposite prediction follows from the above analysis. Niche parties should gain votes when mainstream parties choose dismissive strategies (i.e., under the majority runoff) as in those circumstances their presence doesn't have *adverse* policy effects on them or their followers.

4.2 Majority run-off w/vote thresholds

Some majority run-off systems do not restrict to the second ballot to the top two contenders but allow any party that clears a given vote threshold to advance onto the second ballot. Examples of such systems are the systems used for legislative elections in France and in Hungary (for the seats filled in single-member districts). In France the current threshold is 12.5% of registered voters, i.e., the actual vote share needed to advance will depend on the turnout in the district.[5] Typically, turnout in French legislative elections is around 66% so in effect a party must win 18-19% of the vote to advance. Hungary, on the other hand, uses a threshold rule that considers both the rank of the candidate and her vote share. The three candidates that receive the most vote advance onto the second ballot in addition to any candidate that receives more than 15% of the vote.

We denote the vote threshold by τ. Majority runoff elections with vote thresholds greater than $\frac{1}{3}$ are equivalent to a top-two runoff so we restrict our attention to $\tau \leq \frac{1}{3}$. In discussing the equilibrium of in the top-two runoff we noted that the expressive party played absolutely no role and the equilibrium corresponded exactly with the equilibria of the standard two candidate model. Why might one expect the results to be any different when the threshold is defined in terms of vote shares? The reason is that the prospects of the candidate whose ideal policy lies in between the other two candidates depend on whether the expressive candidate advances onto the second ballot or not. Thus, the candidate may want to choose his platform so as to deny the expressive candidate the chance of advancing. Doing so requires the can-

[5] More accurately, perhaps, the threshold is defined in terms of number of votes (which doesn't vary with turnout).

didate to position himself closer to the expressive candidate and further away from the median position, which in turn allows the candidate at the other end of the political spectrum greater latitude in choosing his own platforms.

Proposition 8 *Suppose $x^1 \leq \frac{1}{2}$, $x^2 \geq \frac{1}{2}$, and $\tau \in (\frac{1}{6}, \frac{1}{2})$. Any strategy profile $p^* = (0, p_1^*, p_2^*)$ such that $p_1^* \in [2\tau - \varepsilon, 2\tau] \cup (1 - p_2^*, 1]$ and $p_2^* \in (1 - 2\tau, 1 - 2\tau + \varepsilon)$ is a ε-Nash equilibrium of the game. The equilibrium policy outcome equals p_2^*.*

Proof: When the candidates play p^*, candidates 1 and 2 advance onto the second ballot where candidate 2 wins a majority of the vote. Candidate 0's strategy is optimal as she receives her maximal payoff. Any deviation $p_1' < p_1^*$ leaves candidate 1's vote share on the first ballot unaltered, $v_1^1 = \frac{p_2^*}{2}$, and reduces the candidate's vote share to $v_1^2 = \frac{p_1' + p_2^*}{2} < \frac{p_1^* + p_2^*}{2}$. Any deviation $p_1' \in (p_1^*, p_2^*)$ means that candidate 0 receives more than τ and advances onto the second ballot. Candidate 1's vote share on the second ballot is then $v_1^2 = \frac{p_2^*}{2}$. Candidate 2 receives $v_2^2 = 1 - \frac{p_1' + p_2^*}{2}$ of the vote. Since candidate 1's vote share will always be larger than candidate 0's vote share $\left(v_0^2 = \frac{p_1'}{2} < v_1^2 = \frac{p_2^*}{2}\right)$, candidate 1's decision hinges on whether choosing a platform close to p_2^* will reduce candidate 2's vote share sufficiently for candidate 1 to win. Such profitable deviation exists if there exist $p_1' \in (p_1^*, p_2^*)$ such that $v_1^2 = \frac{p_2^*}{2} < 1 - \frac{p_1' + p_2^*}{2}$. The inequality reduces to $p_1' > 2 - 2p_2^*$ or, as $\varepsilon \to 0$, $p_1' > 2 - 2(1 - 2\tau) = 4\tau$. Since we are considering deviation such that $p_1' < p_2^*$ then it must be the case that $\tau < 1 - 2\tau$, or $\tau < \frac{1}{6}$. Hence, no such deviation exists. Finally, there are no profitable deviation such that $p_1' \geq p_2^*$. Either such deviation will not alter the outcome or candidate 1 win will but then $u_1(p_1') < u_1(p_2^*)$. Candidate 1's strategy is thus optimal. Candidate 2 wins the contest, and is the rightmost candidate, so there are clearly no profitable deviations such that $p_2' < p_2^* - \varepsilon$. Neither are there any profitable deviations such that $p_2' > p_2^* + \varepsilon$. By the conditions on p_1^* and p_2^* then imply that $\frac{p_1^* + p_2'}{2} > .5$, i.e., candidate 1 receives the vote of the median voter on the second ballot and implement the policy p_1^* but $u_2(p_1^*) < u_2(p_2^*)$. □

Proposition 8 demonstrates how, for given preference configurations, the presence of an expressive candidate prevents median convergence from taking place as occurs under the top-two runoff (given identical preferences). Figure 4 depicts an equilibrium under the conditions of proposition 8. The presence of the expressive candidate works to the advantage of the candidate at the other end of the political spectrum. However, the advantage is smaller than under FPTP because the centrist candidate (candidate 1) can choose a policy platform that ensures that she faces candidate 2 on the second ballot and, therefore, candidate 2 must adopt a platform closer to the median that ensures her a majority on the second ballot. Thus, to a certain degree, a majority runoff with vote share thresholds produces a policy outcome that lies in between the policy outcomes under FPTP and top-two majority runoff elections.

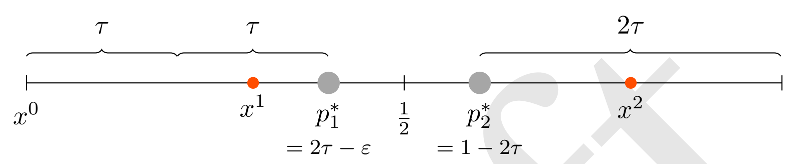

FIGURE 4: EXAMPLE OF AN EQUILIBRIUM (PROPOSITION 8)

Deriving the equilibrium outcomes for the other possible preference configurations is straightforward. Rather than state the propositions formally the equilibria are listed in table 1. Proofing that these are the equilibria outcomes is left as an exercise for the reader.

TABLE 1: EQUILIBRIUM STRATEGIES AND POLICY OUTCOMES UNDER THE MAJORITY RUNOFF WITH A VOTE SHARE THRESHOLD
(AS $\varepsilon \to 0$)

x^1	x^2	p_1^*	p_2^*	p	*if*
$< \frac{1}{2}$	$> \frac{1}{2}$	2τ	$\min\{1-2\tau, \frac{2}{3}, x^2\}$	p_2^*	
$< \frac{1}{2}$	$< \frac{1}{2}$	$[0, x^2)$	x^2	x^2	
$> \frac{1}{2}$	$< \frac{2}{3}$	$[0, x^2)$	x^2	x^2	
$> \frac{1}{2}$	$> \frac{2}{3}$	$[0, \frac{1}{3}) \cup (2\tau, x^2)$	$\frac{2}{3}$	$\frac{2}{3}$	$2\tau < 2x^1 - \frac{2}{3}$
—	—	$[0, 1-x^2) \cup (2\tau, x^2)$	$2x^1 - 2\tau$	$2x^1 - 2\tau$	$2\tau \geq 2x^1 - \frac{2}{3}$
$> \frac{2}{3}$	$> \frac{2}{3}$	x^1	$[0, 1-x^1) \cup (x^1, 1]$	x^1	

Figure 5 graphs the policy outcomes for any pair of the candidates' ideal policies when the vote share threshold equals 20%. The figure is somewhat 'messier' than the corresponding figures (**??**) for the policy outcomes under FPTP with and without (which is also equivalent to the outcomes for the top two run-off) an expressive candidate. Comparing the figures, darker shades indicate policies further to the right, suggests that policy outcomes under the majority run of with a vote threshold are more similar to the outcome under FPTP with an expressive candidate even when the threshold is fairly high. This stands to reason. The presence of an expressive candidate, and the possibility that she may advance onto the second ballot, ensures that the parties must take account of the candidate's presence. As before the response is to adopt policy platforms further away from expressive candidates.

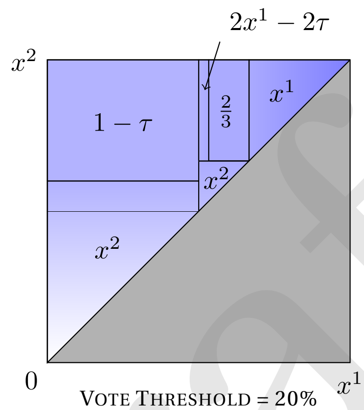

FIGURE 5: POLICY OUTCOMES: MAJORITY RUNOFF WITH A VOTE THRESHOLD (AS $\varepsilon \longrightarrow 0$ FOR ε-NASH EQUILIBRIA)

5 Conclusions

In this paper we have considered how the presence of a non-competitive, expressive candidate influences electoral competition among policy-seeking candidates. The result suggest that expressive candidates generally hurt themselves by contesting elections – although this obviously requires us to attribute some preferences over policy to the candidates. We also considered how the rules of the game influence candidate positioning by comparing FPTP systems with the two major types of majority runoff systems. The comparison shows that the adverse effects of contesting elections for expressive candidates depend on the electoral system. The top-two runoff is shown to have no such adverse effects while systems employing vote thresholds do mitigate such adverse effect (in comparison with FPTP) but do not eliminate them.

We suggest that these factors may help explain the success of extremist parties – extremist parties may be less likely to contest elections if they are likely to lead to shifts in policy away from their preferred position.

The model, as it stands, rests on fairly restrictive assumptions but they can easily be relaxed. Allowing for less extreme expressive candidates, a mixture of policy and office motives, and different distributions of voter ideal points would make the model far more general. However, as I argue above, each of these factors is unlikely to change the substantive results although each of them may work to soften the contrast between contests with and without expressive candidates.

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