Optimal Campaign Spending in Elections

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Abstract

We present a theory of campaign spending in elections. In our model we show that in the absence of competitive electoral pressure the timing of campaign spending will simply on the relative benefits of spending money early vs. late in the campaign. When the candidate have to compete for funds, and their ability to raise funds depends on their standing in the polls, candidates are forced to increase their spending early on in the campaign. This finding appears consistent, e.g., with a number of presidential primary races.

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1 Introduction

Campaign finance is a much discussed topic in the study of politics as well as in the practice of politics. Yet we lack a good theory of campaign spending. Most of the research on campaign spending has focused on campaign contributions, their effect, and the candidates' ability, or inability, to outspend their opponents. Political scientists have paid much less paid attention to how politicians use their resources and the dynamics of raising and spending campaign contributions. There are, of course, a few exceptions. Box-Steffensmeier (1996), Epstein & Zemsky (1995), Goodliffe (1999), and Squire (1991) considers how an incumbent's warchest or fundraising efforts may deter the entry of a challenger. Goodliffe (2001), similarly, considers the role of campaign spending in sequential elections in the same context. Endersby & Petrocik (2001), taking a different approach, examine the effectiveness of the different types of campaign spending, such as direct mailing, canvassing, etc. on candidates' support and electoral outcomes.

Focusing on competition in multiple districts Brams & Davies (1982) and Snyder (1989, 1990) consider how parties and presidential candidates, respectively, allocate their resources between districts. The models provide interesting insights into resource allocation in campaigns but perhaps tell us little about individual single-district campaigns in which the candidates running for office, rather than their party, are responsible for conducting and financing their campaign for the most part. Another curious feature of these model is the presence of a cost that the candidates or parties face in spending their resources.[1] It is unclear to us, and it therefore makes us a little uneasy, that candidates face such cost in spending their resources and contributions, especially given the fact that in the U.S. candidates can not use donations for non-political purposes. Further, research on congressional campaigns, for example, reveals that the amount of personal resources spent by candidates is minuscule compared to the total cost of their campaigns. Finally, we feel it is essential to investigate how much leverage we can get on the issue of how candidates allocate resources over the course of a campaign by sticking with the Mayhewian assumption that candidates for office are single minded seekers of election.

The literature on contests and race within the economics discipline, which occasional makes references to political contests, generally relies on similar assumptions about (what would be) campaign spending in their models. While we certainly agree that there are some strong similarities between the the question we pose in this paper and the contests literature, e.g. Baik & Shogren (1992), Dixit (1987), Harris & Vickers (1985, 1987), and Lee & Wilde (1980), we believe that certain differences between the economic and political realms prohibit the straightforward application of these models. The first reason, the absence of an opportunity cost to for expending effort in a cam-

[1] Snyder (1989) claims that his results go through when the cost of spending and raising money is simply replaced by a budget constraint.

paign has already been mentioned. The second reason regards the financing of political campaigns. In the economic realm, or at least in the models of it, capital markets are assumed to be fluid and in principle each competitor is equal. In the political realm, however, competition for the limited amount of funds is fierce. **?** goes as far as to suggest that congressional candidates are essentially engaged in two distinct contest. One contest involves obtaining resources for their campaigns, the second contest for voters's approval. The two contests are, however, intricately linked and it is this link that we focus on here. This argument has a natural extension beyond congressional elections to other political contests such as primary and presidential elections.

The interdependency between the two contests has been noted by many but modelled by few. Aldrich (1980) provides a model of the dynamics of fundraising and the candidates' electoral strength where fundraising increases electoral success and electoral success increases the candidates' fundraising ability. Aldrich's model is, however, simply based on difference equations with no strategic considerations on behalf of the candidates. While the evidence on electoral campaigns seems to suggest that both of Aldrich's assumption may be correct the model risks missing much of what is interesting about political contests. Meirowitz & Wiseman (2000) present a model similar in spirt using the idea of network externalities to explaining why contributors have an incentive to coordinate their donations on a single candidate.[2] The similarities between the two models lie in the fact that in them the candidates really have no role and the model provide little intuition about what factors might jump-start the types of dynamics described. This is not to say that the models do not produce valuable insights. On the contrary, the models do identify important qualities of electoral campaigns and in some ways we have incorporated these insights into our models in the form of assumptions about contributor and voter behavior.

The question we ask in this paper is *when* do candidates choose to spend their resources and, in particular, how are candidates' spending strategies influenced by the pressure to raise funds for their campaigns. Below we attempt to model some of the important aspects of electoral contest. First, we incorporate the two established observations about campaign discussed above - spending is assumed to increase the likelihood of winning an election and a good standing in the polls, or early primaries, is assumed to help fundraising efforts. But as we have argued there is more to elections than the dynamics embedded in these assumptions. Generally speaking, every electoral contest forces candidates to make numerous decisions regarding the allocation of their resources – whether time or money. Candidates must, for example, decide how much effort is spent soliciting contributions and how much time is spent canvassing voters. Similarly, candidates must decide how to make their appeal to the voters. Canvassing, direct mailing, and radio/tv advertisements are

[2]It should be noted that in Meirowitz and Wiseman's model the contributors are fully rational. Network externalities refer (in this context) to the increase in utility a contributor receives from other contributors donating to the same candidate.

some of the options open to the candidates. In this paper we focus on a particular decision that each candidate faces. This decision concerns the timing of his campaign spending when the candidate must simultaneously engage in competition for campaign funds.

Our results can be summarized as follows.

- When candidates are relatively symmetric with respect to resources and initial level of support, and there is a large pool of donations to contest for, both candidate spend heavily in the early and late stages of the campaign. Think: Clinton vs. Lazio.
- When one candidate has significant advantage over the other with respect to initial level of support, and the other has a relatively small amount of resources at the beginning of the campaign, we find early campaign spending is relatively small (non-existent) compared to late campaign spending. Think: Strong Incumbent vs. Weak Challenger.

2 A Model of Campaign Spending

We model the dynamics discussed in the previous section as follows. Two candidates, A and B, must conduct a campaign for a single office. We assume each candidate's objective is to maximize their probability of winning it. Let $x^E \in \mathbb{R}$ be a variable that summarizes the level of support for candidate A at the end of the campaign. Assume A's probability of winning is a strictly increasing function of x^E. Naturally, B's probability of winning is a strictly decreasing function of x^E.

The campaign begins with an initial level of support for candidate A, denoted by x°, and an initial endowment of resources for each candidate, where i's endowment is denoted by k_i, $k_i \geq 0$. There are two stages to the campaign, stage 1 and stage 2. We assume candidates can influence x^E through spending resources in stages 1 and 2. Let s_i^t denote candidate i's spending in stage t. Formally, x^E can be viewed as function that maps the spending of each candidate over both stages of the campaign into a real number. In what follows

$$x^E((s_A^1, s_A^2), (s_B^1, s_B^2)) = x^{\circ} + \underline{\mu}(s_A^1 - s_B^1) + \overline{\mu}(s_A^2 - s_B^2),$$

where we assume $\underline{\mu} < \overline{\mu}$. Thus, candidate A's support at the end of the game is strictly increasing in his own spending, and strictly decreasing in B's spending. The assumption that $\underline{\mu} < \overline{\mu}$ implies that a unit of resource spent in stage 2 has a greater direct effect on A's final level of support than a unit of resource spent in stage 1. This assumption is intended to capture the notion that the candidates can refine their message over the course of a campaign.[3] While it is perhaps unlikely

[3]For an example of this phenomena, one can track the evolution of slogans employed by George W. Bush in his campaign for president. After losing the New Hampshire Primary to John McCain, the Bush team switched their campaign theme from "Compassionate Conservative" to "Reformer with Results" in an attempt to capture part of the apparent appeal of McCain's reform agenda.

that a candidate is strictly better off spending all his resources in the second period the assumption is innocuous as, if anything, it ought to induce the candidates to spend their resources at the end of their campaigns. We show, however, that competing for contributions creates strong incentives for the candidates to spend money early.

As pointed out in the introduction, a candidate's ability to raise campaign contributions depends, at least in part, on the viability of their candidacy. To capture this aspect of reality, we assume there is a continuum of contributors; denote the set of contributors by C. In our model, the contributors determine a candidate's viability by simply looking at A's level of support at the end of stage 1, x^1, where $x^1(s_A^1, s_B^1) = x^\circ + \underline{\mu}(s_A^1 - s_B^1)$. Contributors make their donations to the campaigns of candidates A and B after the the candidates' stage 1 spending decisions and prior to their stage 2 spending decisions. A contributor $c \in C$ contributes m units of resources to A and 0 units to B if $x_c \leq x^1$, otherwise c contributes m units to B and 0 units to A.[4] The distribution of *contributor thresholds* x_c on $\mathbb{R}$ is given by the distribution function $F(x)$. We assume that F is twice continuously differentiable, and that f, the probability density function associated with F, is single peaked. Thus, conditional on x^1, the amount of contributions A receives is $m \cdot F(x^1)$, and the amount B receives is $m \cdot (1 - F(x^1))$.

Finally, we require that each candidate's spending in each period not exceed his cash on hand. Thus, in the first stage of the campaign candidate i may not spend more than k_i. In the second stage, conditional on s_A^1 and s_A^2, candidate A may not spend more than $k_A - s_A^1 + m \cdot F(x^1(s_A^1, s_B^1))$, and candidate B may not spend more than $k_B - s_B^1 + m \cdot (1 - F(x^1(s_A^1, s_B^1)))$.

To summarize, the candidates determine their campaign spending over the two stages of the campaign. Between the two stages, campaign contributions are made contingent on the candidates' standing, thus determining the candidates' ability to spend in the second stage. As spending in the second stage is assumed to have a greater impact on the candidate's level of support at the end of the game, each candidate confronts tradeoff between spending in the first stage and second stage.

It would be natural to represent the above model as as two–period extensive form game and explore its subgame perfect equilibrium. However, given our assumptions about the objectives of the candidates, it would be a dominant strategy in the second stage of the game for each candidate to spend all of their cash on hand. Why? Recall that x^E is strictly increasing in A's spending, and strictly decreasing in B's spending, thus in stage 2, each candidate would spend all the money left in their campaign war chest. As a result, all of the strategic interaction in the two–period extensive from game representation of our model would occur in its first stage. Therefore, we can capture all the important elements of the above model by representing it as a simultaneous move game

[4] An extreme value of x_c can be interpreted to mean that contributor c places more weight on his desire to contribute to a candidate whose policy position he agrees with than his desire to contribute to the expected winner of the campaign. The opposite interpretation can be attributed to a moderate value of x_c.

$\Gamma(\nu) = \langle \{A, B\}, (S_i(\nu)), (u_i(\nu))\rangle$, $\nu = (k_A, k_B, \underline{\mu}, \overline{\mu}, d, x^\circ, F)$, where the action sets of A and B, and their preferences over action profiles are specified as follows.

Candidate i's action set S_i is $[0, k_i]$. Let s_i denote an arbitrary element of this set. In terms of the model described above, s_i is simply i's spending level in the first stage. Candidate A's preferences over action profiles $(s_A, s_B) \in [0, k_A] \times [0, k_B]$ are represented by a utility index u_A, where

$$u_A(s_A, s_B) \equiv x^E((s_A, k_A - s_A + m \cdot F(x^1(s_A, s_B))), (s_B, k_B - s_B + m \cdot (1 - F(x^1(s_A, s_B))))). \quad (1)$$

Note, to embed the logic of subgame perfection into the simultaneous move representation of our model, the utility candidate A receives from first stage spending levels (s_A, s_B) is equal to his level of support at the end of the campaign x^E when

$$\begin{aligned} s_A^1 &= s_A, \\ s_B^1 &= s_B, \\ s_A^2 &= k_A - s_A^1 + m \cdot F(\underline{\mu}(s_A^1 - s_B^1)) \text{ and,} \\ s_B^2 &= k_B - s_B^1 + m \cdot (1 - F(\underline{\mu}(s_A^1 - s_B^1))). \end{aligned}$$

Candidate B's preferences over action profiles is given by utility index u_B where $u_B(s_A, s_B) = -u_A(s_A, s_B)$.

The solution concept we employ for this game is Nash equilibrium. A pair of spending levels (s_A^*, s_B^*) is a Nash equilibrium to $\Gamma(\nu)$ if and only if $u_A(s_A^*, s_B^*) \geq u_A(s_A, s_B^*)$ for all $s_A \in [0, k_A]$, and $u_B(s_A^*, s_B^*) \geq u_B(s_A^*, s_B)$ for all $s_B \in [0, k_B]$.

3 Equilibria of the Game

In the previous section, we developed a model intended to capture the trade off between spending money early and late in a campaign. We begin this section by discussing at some length the best response functions of each candidate. The purpose of the discussion is two–fold. First, we hope to give the reader an intuition for the incentives candidates confront in our model. Second, it will allow for an efficient characterization of the equilibria of the game. Following our discussion of each candidates' best rest response function, we then establish the following sequence of results. We first establish that there exists a mixed strategy equilibrium to $\Gamma(\nu)$ for any ν that satisfies the assumptions of our model. We then show that in any pure strategy equilibrium of $\Gamma(\nu)$ where one of the candidates spends a positive amount in stage 1, it must be the case that both candidates spend their entire endowment in stage 1. As a corollary to this result, we note that the strategy profile where neither candidate spends a portion of their endowment in the first stage is the only other potential pure strategy equilibrium to $\Gamma(\nu)$. Next, we characterize the set of parameterizations of our model where both candidates spend their entire endowment in the first stage, where neither candidate spends any of their endowment in the first stage, and where both candidate use a mixed strategy.

3.1 Best Response Functions of the Candidates

In deriving the candidates best responses we focus our attention on candidate A. The derivation of candidate B's best response correspondence follows an identical process. For each level of spending in the first stage of the campaign by candidate B, A's best response correspondence, b_A, identifies the set of first stage spending levels that maximize his utility. Formally,

$$b_A(s_B) \equiv \arg\max\{u_A(s_A, s_B) : s_A \in [0, k_A]\}.$$

Thus, $\hat{s}_A \in b_A(s_B)$ if and only if $u_A(\hat{s}_A, s_B) \geq u_A(s_A, s_B)$ for all $s_A \in [0, k_A]$. As a result, $b_A(s_B)$ is the set of solutions to the following inequality–constrained optimization problem:

$$\max u_A(s_A, s_B) \text{ subject to } s_A \geq 0 \text{ and } k_A - s_A \geq 0. \tag{2}$$

We can apply the Kuhn-Tucker theorem to identify the first-order necessary conditions for $s_A \in b_A(s_B)$.[5] Begin by formulating the Lagrangian, L_{s_B}, for A's optimization problem given B's spending level s_B.

$$L_{s_B}(s_A, \lambda_{A1}, \lambda_{A2}) = u_A(s_A, s_B) + \lambda_{A1} s_A + \lambda_{A2}(k_A - s_A),$$

[5] Chapter 6 of Sundaram's (1996) *Optimization Theory* provides a nice discussion of inequality-constrained optimization problems.

where

$$u_A(s_A, s_B) = x^\circ + \underline{\mu} s_A - \underline{\mu} s_B + \overline{\mu}[(k_A - s_A + mF(x^\circ + \underline{\mu} s_A - \underline{\mu} s_B)) \\ -(k_B - s_B + m(1 - F(x^\circ + \underline{\mu} s_A - \underline{\mu} s_B)))]. \tag{3}$$

The Kuhn-Tucker theorem implies that if $s_A \in b_A(s_B)$, then there exists a $(s_A, \lambda_{A1}, \lambda_{A2})$ that is a solution to the the following set of equations:

$$\frac{\partial L_{s_B}}{\partial s_A}(s_A, \lambda_{A1}, \lambda_{A2}) = \underline{\mu} - \overline{\mu} + 2\overline{\mu} m f(x^\circ + s_A^1 - s_B^1)) + \lambda_{A1} - \lambda_{A2} = 0 \tag{4}$$
$$\frac{\partial L_{s_B}}{\partial \lambda_{A1}}(s_A, \lambda_{A1}, \lambda_{A2}) = s_A \geq 0,\ \lambda_{A1} \geq 0,\ \lambda_{A1} s_A = 0$$
$$\frac{\partial L_{s_B}}{\partial \lambda_{A2}}(s_A, \lambda_{A1}, \lambda_{A2}) = k_A - s_A \geq 0,\ \lambda_{A2} \geq 0,\ \lambda_{A2}(k_A - s_A) = 0.$$

A solution to the above set of equations shall be referred to as a critical point of L_{s_B}.

For sake of notational simplicity we will normalize the effect of spending in the first period to 1 and denote $\frac{\overline{\mu}}{\underline{\mu}}$ as simply μ. The normalization, and a bit of rearranging of equation 4, then gives us:

$$2\mu m f(x^\circ + s_A^1 - s_B^1) = \mu - 1 - \lambda_{A1} + \lambda_{A2} \tag{5}$$

If there exists a critical point to L_{s_B} where $s_A \in (0, k_A)$, then equation 5 can be re-written as

$$2\mu m f(x^\circ + s_A^1 - s_B^1) = \mu - 1 \tag{6}$$

Equation 6 has an intuitive interpretation. The LHS of the equation 6 corresponds to the marginal benefit of spending in the first period, i.e., the effect it has on the contributions to the candidate and their value in the second period. The RHS of equation 6, on the other hand, corresponds to the marginal benefit of conserving your funds for use in the second period – the value of doing so is simply the difference between the marginal effect of spending on popularity in the two periods. Thus, when $b_A(s_B) \in (0, k_A)$, the marginal benefit to candidate A of spending in the first stage must be equal to the marginal benefit of A conserving his funds for the second stage.

In the rest of the paper the form of equation 6 we will work with, for reasons that will become clear, is the following:

$$f(x^\circ + s_A^1 - s_B^1) = \frac{\mu - 1}{2\mu m} \tag{7}$$

The solutions to equation 7 can be represented graphically – see Figure 1. In Figure 1 $f(x)$ is the probability density function of contributor thresholds. As $z(x; \mu, m)) \equiv \frac{\mu-1}{2\mu m}$ does not depend on x, it is simply a horizontal line. For all $x \in \mathbb{R}$ where $f(x)$ and $z(x)$ intersect, given x° and s_B, there exists a $s_A \in \mathbb{R}$ that satisfies equation 7. Those $s_A \in [0, k_A]$ are the only possible solutions to equation 7 that solve equation 2.

The inverse of $f(x)$, $f^{-1}(x)$, is a correspondence that takes two values by our assumptions about $f(x)$.[6] Let f^{-1}_{max} be the $x \in \mathbb{R}$ such that $f(x) \geq f(y)$ for all $y \in \mathbb{R}$. Let $\underline{x} \equiv \min f^{-1}(z)$ and let $\overline{x} \equiv \max f^{-1}(z)$. As a result, there are only two potential interior critical points to L_{s_B}. These points are: $(\overline{x} - (x^\circ - s_B), 0, 0)$ and $(\underline{x} - (x^\circ - s_B), 0, 0)$. One can check that both of these points are solutions to equation 7. A sufficient condition for an interior critical point of L_{s_B} to be a local maximum of L_{s_B} is that its second derivative is negative. Note that,

$$\begin{aligned} &2\mu m f'(x^\circ + s_A^1 - s_B^1) < 0 \quad if \quad x^\circ + s_A^1 - s_B^1 > f^{-1}_{max} \\ &2\mu m f'(x^\circ + s_A^1 - s_B^1) > 0 \quad else \end{aligned} \tag{8}$$

Thus, $\underline{x} - (x^\circ - s_B)$ is a local minimum of L_{s_B}, and as a result is not among A's best responses to s_B. However, $\overline{x} - (x^\circ - s_B)$ is a local maximum of L_{s_B}. As a result, $b_A(s_b) \subset \{0, \overline{x} - (x^\circ - s_B), k_A\}$. Due to the importance of this result, we state it as a lemma.

Lemma 1 $b_A(s_B) \subset \{0, \overline{x} - (x^\circ - s_B), k_A\}$

The following lemma characterizes the best response function for candidate A. These results can be formally established by using the techniques employed in inequality-constrained optimization problems.

Lemma 2 *Consider the simultaneous move game* $\Gamma(v)$.

a. *If* $x^\circ - s_B \geq \overline{x}$, *then* $b_A(s_B) = 0$.

b. *If* $x^\circ - s_B \in [\underline{x}, \overline{x})$, *then* $b_A(s_B) = \min\{k_A, \overline{x} - (x^\circ - s_B)\}$.

c. *If* $x_\circ - s_B < \underline{x}$, *then*

$$b_A(s_B) = \begin{cases} 0 & \textit{if } u_A(\min\{k_A, \overline{x} - (x^\circ - s_B)\}, s_B) < u_A(0, s_B) \\ \{0, \min\{k_A, \overline{x} - (x^\circ - s_B)\}\} & \textit{if } u_A(\min\{k_A, \overline{x} - (x^\circ - s_B)\}, s_B) = u_A(0, s_B) \\ \min\{k_A, \overline{x} - (x^\circ - s_B)\} & \textit{otherwise} \end{cases}.$$

The following lemma characterizes the best response function of candidate B. Its logic is parallel to the previous lemma.

Lemma 3 *Consider the simultaneous move game* $\Gamma(v)$.

a. *If* $x^\circ + s_A \leq \underline{x}$, *then* $b_A(s_B) = 0$.

b. *If* $x^\circ + s_A \in (\underline{x}, \overline{x}]$, *then* $b_B(s_A) = \min\{k_B, x^\circ + s_A - \underline{x}\}$.

[6] With the exception of the x corresponding to the peak of $f(x)$.

c. *If $x^\circ + s_A > \overline{x}$, then*

$$b_B(s_A) = \begin{cases} 0 & \textit{if } u_B(s_A, \min\{k_B, x^\circ + s_A - \underline{x}\}) < u_A(s_A, 0) \\ \{0, \min\{k_B, x^\circ + s_A - \underline{x}\}\} & \textit{if } u_B(s_A, \min\{k_B, x^\circ + s_A - \underline{x}\}) = u_A(s_A, 0) \\ \min\{k_B, x^\circ + s_A - \underline{x}\} & \textit{otherwise} \end{cases}.$$

3.2 Characterization of Equilibrium

Say an equilibrium (s_A^*, s_B^*) to $\Gamma(\nu)$ is a *full-spending* equilibrium if $s_A^* = k_A$ and $s_B^* = k_B$. Say an equilibrium (s_A^*, s_B^*) to $\Gamma(\nu)$ is a *no-spending* equilibrium if $s_A^* = 0$ and $s_B^* = 0$. In this section, we establish that if a pure strategy equilibrium exists to $\Gamma(\nu)$, then it is one of these two types. Finally, we show that there always exists mixed strategy equilibrium to $\Gamma(\nu)$.

Proposition 1 *Assume (s_A^*, s_B^*) is a pure strategy Nash equilibrium of $\Gamma(v)$. If $s_A^* > 0$ or $s_B^* > 0$, then $s_A^* = k_A$ and $s_B^* = k_B$.*

Proof: Assume (s_A^*, s_B^*) is a pure strategy Nash equilibrium of $\Gamma(v)$. Without loss of generality, assume $s_A^* > 0$. We need to show $s_A^* = k_A$, and $s_B^* = k_B$. Suppose, to the contrary, $s_A^* \neq k_A$ or $s_B^* \neq k_B$. Then it follows that we fall into one of two cases: (i) $s_A^* \in (0, k_A)$ and $s_B^* \in [0, k_B]$ or (ii) $s_A^* = k_A$ and $s_B < k_B$. We shall now show that each case will lead to a contradiction.

Consider case i: $s_A^* \in (0, k_A)$ and $s_B^* \in [0, k_B]$. By assumption, (s_A^*, s_B^*) is a Nash equilibrium of $\Gamma(v)$, thus $s_A^* \in b_A(s_B^*)$ and $s_B^* \in b_B(s_A^*)$. By lemma 1, $b_A(s_B^*) \subset \{0, \overline{x} - (x^\circ - s_B^*), k_A\}$. Since we assumed that $s_A^* > 0$ and $s_A^* < k_A$, $s_A^* = \overline{x} - (x^\circ - s_B^*)$.

Suppose $s_B^* = 0$, then $x^\circ + s_A^* = \overline{x}$. However, by part *b* of Lemma 3, $b_B(s_A^*) > 0$. Thus, $s_B^* \notin b_B(s_A^*)$, a contradiction. Suppose $s_B^* \in (0, k_B)$. Then $x^\circ + s_A^* > \overline{x}$. However, by party *c* of Lemma 3, if $s_B^* \in b_B(s_A^*)$ then $b_B(s_A^*) \subset \{0, \min\{k_B, x^\circ + s_A^* - \underline{x}\}$. However, $x^\circ + s_A^* - \underline{x} = \overline{x} - \underline{x} + s_B^*$. Since $\overline{x} - \underline{x} > 0$, it follows that $s_B^* \notin b_B(s_A^*)$, a contradiction. Suppose $s_B = k_B^*$. This implies $f(x^\circ + s_A^* - s_B) \leq z$ for all $s_B \in [0, k_B]$. This implies that $u_B(s_A^*, 0) > u_B(s_A^*, k_A)$. As a result, it follows from part c of Lemma 3, that $b_B(s_A) = 0$. Thus, $s_B^* \notin b_B(s_A^*)$, a contradiction.

Consider case ii: $s_A^* = k_A$ and $s_B^* < k_B$. By assumption, (s_A^*, s_B^*) is a Nash equilibrium of $\Gamma(v)$, thus $k_A \in b_A(s_B^*)$ and $s_B^* \in b_B(s_A^*)$.

First, note that $b_A(s_B) = k_A$ implies $x^\circ + k_A - s_B^* \in (\underline{x}, \overline{x}]$. Suppose not. Then either $x^\circ + k_A - s_B^* \leq \underline{x}$ or $x^\circ + k_A - s_B^* \leq \underline{x}$. If the former is the case, then it can be shown that $b_A(s_B) = 0$. Thus, $k_A \notin b_A(s_B^*)$, a contradiction. If the latter is the case, then either $x^\circ - s_B^* > \overline{x}$ or $x^\circ - s_B^* < \overline{x}$. If $x^\circ - s_B^* > \overline{x}$, then part a of Lemma 2 implies that $b_A(s_B) = 0$. Thus $k_A \notin s_A(s_B^*)$, a contradiction. If $x^\circ - s_B^* < \overline{x}$, then $\min\{k_A, \overline{x} - (x^\circ - s_B^*)\} = \overline{x} - (x^\circ - s_B^*)$. As a result, parts *b* and *c* of Lemma

2 imply that $b_A(s_B^*) \in \{0, \overline{x} - (x^\circ - s_B^*)\}$. Thus, $k_A \notin b_A(s_B^*)$, a contradiction. Therefore, it must be the case that $x^\circ + k_A - s_B^* \in (\underline{x}, \overline{x}]$.

If $s_B^* = 0$, then by part b of Lemma 3, $b_B(s_A^*) > 0$. Thus, $s_B^* \notin b_B(s_A^*)$, a contradiction. If $s_B^* \in (0, k_B)$, and $s_B^* \in b_B(k_A)$, then it must be the case that $x^\circ + s_A^* + s_B^* = \underline{x}$. However, this implies that $k_A \notin b_A(s_B^*)$, a contradiction. ■

Corollary 2 *Assume (s_A^*, s_B^*) is a pure strategy Nash equilibrium of $\Gamma(v)$. If $(s_A^*, s_B^*) \neq (k_A, k_B)$, then $s_A^* = 0$ and $s_B^* = 0$.*

This is a simple consequence of the contra-positive of Proposition 1.

Proposition 3 *There exists a mixed strategy equilibrium to $\Gamma(\nu)$*

Proof: It is easily checked that u_A and u_B are both continuous in (s_A, s_B). Further, S_A and S_B are compact sets. Thus, by Theorem 1.3 of Fudenberg and Tirole (1996, 35) we are ensured existence of a mixed strategy equilibrium. ■

Finally, note, there exists parameterizations of our model where u_A is not quasi-concave in s_A. Thus, we can not generally apply standard existence theorems for pure strategy equilibrium to $\Gamma(\nu)$. In the next section, we show it is indeed the case that for some ν, there fail to exist pure strategy equilibrium to $\Gamma(\nu)$.

3.3 Conditions for Existence of Types of Equilibrium

Above it has been shown that the only pure strategy equilibrium that can exist in the game is of one of two types. Either the candidates spend all of their initial endowment or they spend nothing. In the rest of this section we characterize the conditions on the parameters of the model under which each of the equilibria occurs. We first prove a sufficient condition for the existence of no-spending equilibria.

Proposition 4 *A no-spending equilibrium exists if $z \geq f_{max}^{-1}$*

Proof: The proof of the sufficiency of this condition is trivial. If $z > f_{max}^{-1}$ it implies that the marginal benefit of spending in the second period is greater than the marginal benefit of spending in the first period. It can thus never be optimal for either of the candidates to spend any of their resources in the first period. ■

We now turn to the sufficient and necessary conditions for the existence of full-spending pure strategy equilibrium. First, define $z^\star$ as the maximum $z \in \mathbb{R}$ such that both the following conditions hold:

$$F(x^1(k_A,k_B)) - F(x^1(0,k_B)) \geq z(x^1(k_A,k_B) - x^1(0,k_B))$$
$$F(x^1(k_A,0)) - F(x^1(k_A,k_B) \geq z(x^1(k_A,0) - x^1(k_A,k_B))$$

When the first condition holds with equality candidate A is indifferent between spending nothing and spending all of his initial endowment when candidate B is spending all of his initial endowment. The second condition has an analogous interpretation for candidate B. Intuitively, the conditions require each of the candidates to be at least as well off spending all of their resource as spending nothing when their opponent spends his entire endowment.

Proposition 5 *A full-spending equilibrium exists iff $z \leq z^\star$*

Proof: Proposition 1 establishes that in any pure strategy equilibrium both of the candidates spend either all or nothing. To show sufficiency it is therefore enough to show that: *a)* No spending by both candidates is not an equilibrium and *b)* both candidates spending everything is an equilibrium. To show *a)* note that by the definition of $z^\star$, there exists a level of spending for each candidate such that the candidate is better off than by spending nothing when the other candidate spends nothing. To show *b)* first note that at $z \leq z^\star$ it must be the case that $x^1(k_A,k_B) \in (\underline{x},\overline{x})$. To see this fix z and assume without loss of generality that $x^1(k_A,k_B) \geq \overline{x}$. Note that, by definition of $\overline{x}$, $f(x^1(k_A,k_B)) \leq z$ and $f(x^1(k_A,0)) < z$. It follows that $F(x^1(k_A,0)) - F(x^1(k_A,k_B) < z(x^1(k_A,0) - x^1(k_A,k_B))$, contradicting the definition of $z^\star$. Having shown that $x^1(k_A,k_B) \in (\underline{x},\overline{x})$, taking part *a)* of this proof together with parts b and c of Lemmas 2 and 3 demonstrates that the candidates' strategies are indeed optimal at (k_A,k_B). To show the necessity of the condition suppose that $s_A^1 = k_A$ and $s_B^1 = k_B$ are the candidates' equilibrium strategies but that $z > z^\star$ implying that at least one of the defining inequalities of $z^\star$ is not satisfied. Without loss of generality assume $F(x^1(k_A,0)) - F(x^1(k_A,k_B) < z(x^1(k_A,0) - x^1(k_A,k_B))$, implying that $x^E(k_A,k_B) > x^E(k_A,0)$ – i.e. $u_B(k_A,0) > u_B(k_A,k_B)$, thus contradicting the supposition that (k_A,k_B) is a Nash equilibrium. ■

The conditions for the existence of a pure strategy full-spending equilibrium in Proposition 5 are conveniently characterized in terms of z, which is a function of μ and m. If μ is relatively low and m relatively high the candidates will spend all their resources in the first stage of the campaign.

So far we have shown how the existence of no- and full-spending equilibria depends on the level of z. For high levels of z ($\geq f_{max}^{-1}$) no spending occurs but for low levels of z ($\leq z^\star$) the candidates spend all their resources. It should be noted however that the sufficient condition are not the same, i.e., $f_{max}^{-1} \geq z^\star$, therefore raising the question what occurs when $f_{max}^{-1} > z > z^\star$. As Proposition 5 contains both the sufficient and necessary conditions for full-spending equilibria we know that the only equilibria that can arise when $z \in (z^{\star}, f_{max}^{-1})$ are either no-spending equilibria or mixed-strategy equilibria. As it turns out both types of equilibria can be found in this region, thus explaining why the condition of Proposition 4 is merely sufficient. Before proving Proposition 6, let us define x_B^{*}. If there exists a $\hat{x} \in (-\infty, x^{\circ})$ such that

$$F(x^{\circ}) - F(\hat{x}) = \frac{(\mu - 1)(x^{\circ} - \hat{x})}{2\mu m} = z(x^{\circ} - \hat{x}),$$

then let $x_B^{*} = \hat{x}$, otherwise let $x_B^{*} = -\infty$.

Proposition 6 *Assume $z \in (z^{\star}, f_{max}^{-1})$. a) A no-spending "poor man's" equilibrium occurs if $x^{\circ} > \overline{x}$ and $k_B \leq x^{\circ} - x_B^{\star}$. b) Only mixed-strategy equilibria exist if $x^{\circ} < \overline{x}$ or $k_B > x^{\circ} - x_B^{\star}$.*

Proof: Lets begin by proving part *a)*. Suppose $x^{\circ} > \overline{x}$ and $k_B \leq x^{\circ} - x_B^{\star}$. The proof only requires a couple of observations. First, if $x^{\circ} > \overline{x}$ then candidate A has no incentive to campaign in the first period unless candidate B spends more than $x^{\circ} - \overline{x}$ as the marginal value to spending in the first period is lower than the marginal value of saving resources for the second period. This condition is satisfied as candidate B never finds it worthwhile to spend any of his resources when $k_B < x^{\circ} - x_B^{\star}$ as by definition $x^{\circ} - x_B^{\star}$ is the amount of spending candidate must exceed before he realizes a positive effect on his utility. Hence, a no-spending equilibrium occurs.

Now, turn to the proof of part *b)*. Suppose $x^{\circ} < \overline{x}$. Then the strategy profile $(0, 0)$ is not an equilibrium as $f(x^{\circ}) > z$. By $z > z^{\star}$ the full-spending equilibrium does not exist either as shown in Proposition 5. By Proposition 1 these are the only possible pure strategy equilibria. Since no pure strategy equilibrium exist in $\Gamma(\nu)$, if an equilibrium exists to $\Gamma(\nu)$ it must involve mixed strategies. Proposition 3 ensures the existence of mixed strategy equilibria to $\Gamma(\nu)$. Finally, suppose $k_B > x^{\circ} - x_B^{\star}$. Then it can be established using similar arguments that (0,0) is not a pure strategy equilibrium, which therefore implies that the only equilibrium to the game are mixed strategy equilibrium. ■

The sufficient conditions identified for the no-spending equilibria in Proposition 4 and Proposition 6 give rise to different substantive interpretations. In the games in which the no-spending equilibrium described in Proposition 4 occurs, the reason is simply that neither candidate finds raising campaign contributions worth the effort. This might occur because the total amount of contributions is low, or because the marginal effect of campaigning is considerably higher in the second period than in the first. In the "poor man's" equilibrium neither of these reasons need to be true. The first condition for the equilibrium requires the initial popularity of one of the candidates to be considerably higher than that of the other candidate. If the candidate's popularity is sufficiently high, then he has no incentive raise contributions – as long as the other candidate does not either. The second requirement is that the less popular candidate has a relatively low initial endowment of resources. When the candidate has a sufficiently low level of resources it is not worth his while spending any of his resources because for all $s_B \in [0, k_B]$, the cost of spending money in stage 1 is always greater than its benefit. See Figure x for a visual of depiction of this phenomena. A candidate with a sufficiently high initial endowment would, on the other hand, be able to spend enough to overcome this disadvantage. It should be noted that the "poor man's" equilibria does not necessarily imply a lack of resources – only a lack of resources relative to the candidate's initial popularity.

4 Empirical Hypotheses

Propositions 4-6 offer the condition under which no-spending, full-spending, and mixed equilibria occur. These condition also indicate how the various parameters of the model should effect the timing of campaign spending. As we have not yet characterized the mixed-strategy equilibria we can not offer conclusive predictions about the comparative statistics of the model. The preliminary hypothesis/comparative statistics offered here are thus based on our results about the no-spending and full-spending equilibria.

Hypothesis 7 *High amount of total contributions available to the candidates, m, are associated with high levels of early campaign spending.*

Hypothesis 8 *The candidates' ability to refine their message to the taste of the voters, or other means of increasing the relative value of second period spending, is positively associated with high levels of early campaign spending.*

The above hypotheses follow naturally from propositions 5 and 4, as z is decreasing μ and increasing in m. Consider a game in which no spending occurs. Changing the parametrization of that game so that z decreases (by either increasing μ or decreasing m -or both) sufficiently and we can generate an equilibrium in which both candidates spend everything.

Hypothesis 9 *High levels of initial endowment of one or both candidates are associated with lower levels of initial spending.*

Consider two games (add notation) identical in all parameter values save candidate A's initial endowment, which we assume is higher in the second game. Suppose that the parameters of the first game are such that there exists a full-spending equilibrium. By the condition of Proposition 4 a no-spending equilibrium can clearly not exist in the second game. The condition of Proposition 5 may, however, fail as k_A is increased, in which case only a mixed-strategy equilibria can exist.

Hypothesis 10 *Divergence in the candidates' popularity is positively associated with lower levels of spending.*

Lower levels of spending are expected for a similar reason as in observation 9, i.e., the conditions of Proposition 5 will fail for extreme divergence in the candidates' popularity. Note, however, unlike in the previous observation, an increase in divergence may result in a no-spending "poor man's" equilibrium rather than a mixed-strategy equilibrium for some parameter configurations.

5 Conclusions

In future versions of this paper, we aim to do several things. First, for parameterizations of our model where pure strategy equilibrium do not exist, we would like to characterize the properties of the mixed strategy equilibria. This would allow us to derive the comparative statics of the equilibrium to our game. Once the comparative statistics have been obtained we hope to evaluate the model empirically.

The model allows for some extensions. With little difficulty the model could consider contests between candidates of different abilities. Experienced candidates may, for example, be better able to modify their campaign message. Their experience may have taught them how to respond to a variety of challenges that a less experienced candidate struggles with. In our model this difference could be accommodated simply by assuming that the two candidates experience different returns from spending in the second period, i.e., their μ's differ. One can also hypothesize that incumbents have lesser leeway to modify their message over the course of a campaign because they have well established reputations, which could be accommodated in a similar manner.

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